All-Frequency Rendering with Dynamic, Spatially-Varying Reflectance

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Complex, detailed reflectance

• Spatial/temporal variation
• All BRDF types:
  – parametric ↔ measured
  – isotropic ↔ anisotropic
  – glossy ↔ mirror-like
Previous work

Spatial Variation

SVBRDF

single BRDF

Temporal Variation

static
dynamic
Rendering Equation

\[ R(x,o) = \int_{\Omega} L(x,i) \rho(x,i,o) \max(0,n \cdot i) \, di \]
Precomputed Radiance Transfer

• A way to shade objects under different illumination
  – Any kind of light transport is possible
  – Real-time, allowing lighting to change

• Objects have to be static

\[ L_0(p \leftarrow \omega) = L_{env}(\omega)V(p, \omega) \]
Rendering Equation

\[ R(x, o) = \int_{\Omega} L(x, i) \rho(x, i, o) \max(0, n \cdot i) \, di \]

\[ L_0(p \leftarrow \omega) = \frac{L_{\text{env}}(\omega) V(p, \omega)}{\cos(\theta)} \]

\[ R(x, o) = \int_{\Omega} L(i) \rho(x, i, o) V(x, i) \max(0, n \cdot i) \, di \]

- light
- SVBRDF
- visibility
- cosine
Outline

• Reflectance Representation
  – Microfacet Model with SGs

• Visibility Representation
  – Signed Spherical Distance Function

• Lighting & Rendering
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Spherical Gaussian (SG)

\[ G(v; p, \lambda, \mu) = \mu e^{\lambda (v \cdot p - 1)} \]

- trivial rotation
- all-frequency signals

**inner product:**  
\[ G_1 \cdot G_2 = \int_{\Omega} G_1(v) G_2(v) dv = \frac{4\pi \mu_1 \mu_2}{e^{\lambda_1 + \lambda_2}} \frac{\sinh(d_m)}{d_m} \]

**vector product:**  
\[ G_1 \otimes G_2 = G_1(v) \cdot G_2(v) = G\left(v; \frac{p_m}{\|p_m\|}, \lambda_m \|p_m\| \mu_1 \mu_2 e^{\lambda_m(\|p_m\|-1)}\right) \]
SG Mixtures

Sum of Multiple SGs: \( F^* (v) = \sum_{i=1}^{N} G(v; p_i, \lambda_i, \mu_i) \)

Original

SG, \( N = 7 \)

SG, \( N = 3 \)

SG, \( N = 1 \)
Microfacet BRDF Model

- surface modeled by tiny mirror facets

\[
\rho(i, o) = k_d + k_s \rho_s(i, o)
\]

\[
\rho_s(i, o) = M_o(i) D(h)
\]

Normal distribution function
Microfacet BRDF Model

• surface modeled by tiny mirror facets

\[ \rho_s(x,o,i) = \frac{D(x,h)S(x,o)S(x,i)F(x,o,i)}{4(i \cdot n)(o \cdot n)} \]

normal distribution  shadow term  fresnel term  [Cook 82]
Parametric Models

- single-lobe, analytic approximation
  
  – Cook-Torrance [Cook et al. 1981]

\[
\rho_s(i, o) = \frac{F(i, o)S(i, o)}{\pi(n \cdot i)(n \cdot o)} e^{-\left(\frac{\theta}{m}\right)^2}
\]

\[
M_o(i) = \frac{F(i, o)S(i, o)}{\pi(n \cdot i)(n \cdot o)} \left\{ \begin{array}{l} F(o, i) = \frac{(g-o)^2}{2(g+c)^2} \left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right) \\ g = \sqrt{\eta^2 + c^2 - 1}, c = |i \cdot h| \end{array} \right. 
\]

\[
D(h) = e^{-\frac{\arccos(h \cdot n)}{m}} \approx e^{-\frac{2(1-(h \cdot n))}{m^2}} = G(h; n, 2/m^2, 1)
\]
Parametric Models

- single-lobe, analytic approximation


\[
\rho_s(i, o) = \frac{1}{\sqrt{(i \cdot n)(o \cdot n)}} \cdot \frac{e^{-\tan^2 \theta / \alpha^2}}{4\pi \alpha^2}
\]

\[
M_o(i) = \frac{1}{4\pi \alpha^2 \sqrt{(i \cdot n)(o \cdot n)}}
\]

\[
D(h) = e^{-(1-(h \cdot n)^2)) / \alpha^2 (h \cdot n)^2} \approx G(h, n, 2/\alpha^2, 1)
\]
Parametric Models

- single-lobe, analytic approximation

— Blinn-Phong [Blinn 1977]

\[ \rho_s(i, o) = \frac{n + 2}{2\pi} \cos^n \theta \]

\[ M_o(i) = \frac{n + 2}{2\pi} \]

\[ D(h) = (h \cdot n)^n \approx G(h; n, n, 1) \]
Parametric BRDFs

Blinn Phong
Diffuse: (0.17, 0.17, 0.17)
Specular: (68, 68, 68)
Shinness: 1000
Anisotropic Parametric Models

• multi-lobe, analytic approximation

– Ashikhmin-Shirley anisotropic model

[Ashikhmin and Shirley 2000]

\[
\rho_s(i, o) = \frac{\sqrt{(n_u + 1)(n_v + 1)} (\mathbf{n} \cdot \mathbf{h}) n_u \cos^2 \phi_h + n_v \sin^2 \phi_h}{8\pi (\mathbf{h} \cdot \mathbf{i}) \max(\mathbf{i} \cdot \mathbf{n}, \mathbf{o} \cdot \mathbf{n})} \ F(\mathbf{h} \cdot \mathbf{i})
\]

\[
M_o(i) = \frac{\sqrt{(n_u + 1)(n_v + 1)} F(\mathbf{h} \cdot \mathbf{i})}{8\pi (\mathbf{h} \cdot \mathbf{i}) \max(\mathbf{i} \cdot \mathbf{n}, \mathbf{o} \cdot \mathbf{n})}
\]

\[
D(h) = (\mathbf{n} \cdot \mathbf{h}) n_u \cos^2 \phi_h + n_v \sin^2 \phi_h
\]

\[
\theta_v = \arccos \left( \frac{1}{n_v} \right), \quad \theta_i = \frac{2\theta_v}{N + 1}
\]
Anisotropic Parametric Models

$n_u = 8, \ n_v = 128$

$n_u = 25, \ n_v = 400$

$n_u = 75, \ n_v = 1200$
Measured BRDFs

• Isotropic BRDF
  – Using Levenberg-Marquardt optimization
    [Nocedal and Wright 1999]

• anisotropic BRDF
Measured BRDFs

- BRDF from [Matusik03]
- svBRDF from [Wang08] & [Lawance06]
Representation Efficiency

- Parametric BRDF
  Texturing of original BRDF parameters
  isotropic: 7 float/texel: diffuse, specular, shininess
  Anisotropic: 8 float/texel: diffuse, specular, shininess u/v

- Measured BRDF
  Texturing of SGs

<table>
<thead>
<tr>
<th></th>
<th>number of SGs</th>
<th>Floats per SG</th>
<th>floats per texel</th>
</tr>
</thead>
<tbody>
<tr>
<td>isotropic</td>
<td>1-3</td>
<td>4</td>
<td>4~12 + 3</td>
</tr>
<tr>
<td>anisotropic</td>
<td>2-7</td>
<td>6</td>
<td>12~42 + 3</td>
</tr>
</tbody>
</table>
Rendering Equation

\[ R_s(x, o) = \int_{\Omega} L(x, i) \rho_s(x, i, o) \max(0, n \cdot i) \, di \]

\[ \rho_s(i, o) = M_o(i) D(h) \]
BRDF Slices

\[ h = \frac{i + o}{\|i + o\|} \]

\[ i = \Psi(h) = 2(o \cdot h)h - o \]

\[ W(i) = D(\Psi^{-1}(i)) \]
SG Warping

\[ W^*(i) \approx D^*(\Psi^{-1}(i)) \]

- SG not closed under \( \Psi^{-1} \)
- approx. by per-SG warp

\[ p^W = \Psi(p^D) = 2(o \cdot p^D) p^D - o \]
\[ \lambda^W = \lambda^D / \tau_\Psi(p^D) = \lambda^D / \left( \frac{4 \cdot |p^D \cdot o|}{|p^D|} \right) \]
\[ \mu^W = \mu^D \]
\[ W^*(i) = \sum_{i=1}^{n} G(i; p_i^W, \lambda_i^W, \mu_i^W) \approx D^*(h). \]
SG Scaling

• Shadowing and Fresnel terms
  – Assume low-frequency \([\text{Ashkmin01, Ngan05}]\)
  – approx. by per-SG scale

\[
p^\rho = p^W
\]
\[
\lambda^\rho = \lambda^W
\]
\[
\mu^\rho = \mu^W \frac{S(p^W)S(o)F(p^W, o)}{4(p^W \cdot n)(o \cdot n)}
\]
\[
\rho^*_s(i; o) = M_o(i) \otimes W^*(i) \approx \sum_{i=1}^{n} G(i; p_i^W, \lambda_i^W, M_o(p_i^W) \mu_i^W)
\]
Parametric svBRDF Painting
Outline

• Reflectance Representation
  – Microfacet Model with SGs

• Visibility Representation
  – Signed Spherical Distance Function

• Lighting & Rendering
Visibility Representation

- Instead of binary visibility function, using Spherical Signed Distance Function

- Stores signed angular distance

- Why?
  - Ghost-free!
Binary Visibility Function

$V(x, i)$
Spherical Signed Distance Function

\[ V^d(i) = \begin{cases} 
+ \min \arccos(t \cdot i), & \text{if } V(i) = 1 \\
- \min \arccos(t \cdot i), & \text{if } V(i) = 0 
\end{cases} \]
Ghosting artifact in Interpolation
Ghosting-Free Interpolation
Procedure

- Sampling Visibility
  - Per-vertex
- Construct SSDF and Compression
- Reconstruct Visibility per-pixel

Pre-compute

Run-time
Detailed Procedure

1. Sample binary visibility the hemisphere
2. Calculate SSDF
3. Re-parameterize to a square image
4. Compress using PCA

**PCA:**

\[ V^d(x, i) \approx \sum_{j=1}^{M} V^d_j(i) w_j(x) \]

- Eigenvector (stored in texture)
- PCA coefficients (vertex attributes)
Detailed Procedure (Cont’d)

- Reconstruct
  - Calculate inner, vector product with SG

\[
V'(i) = \begin{cases} 
  1, & \delta(i) \geq \frac{\pi}{2} - \theta_d \\
  0, & \text{otherwise}
\end{cases}
\]

\[
\delta(i) = \frac{\pi}{2} - \arctan \left( \frac{i \cdot x^+}{i \cdot z^+} \right)
\]
Detailed Procedure (Cont’d)

• Reconstruct
  – Calculate inner, vector product with SG
Detailed Procedure (Cont’d)

- inner product

\[
G(i; p, \lambda, \mu) \cdot V(i) \approx G(i; p, \lambda, \mu) \cdot V'(i)
= \mu \int_{\delta_0}^{\pi} \int_{0}^{\pi} G(i; z^+, \lambda, 1) \sin \xi \, d\xi \, d\delta
= \mu f_h(\theta_d, \lambda).
\]

\[
\hat{f}_h(\theta_d, \lambda) \approx \sigma(\theta_d, k_\lambda) = \frac{a}{1 + k_\lambda e^{\theta_d}} + \frac{1 - a}{2}.
\]

\[k_\lambda \approx f_k(\lambda) = 0.204\lambda^3 - 0.892\lambda^2 + 2.995\lambda + 0.067.\]

\[a = 1.05\]
Detailed Procedure (Cont’d)

- Vector Product
  - Is a SG

\[ G(i;p, \lambda, \mu) \otimes V(i) \approx G \left( i; p, \lambda, \frac{f_h(\theta_d, \lambda)}{f_h(\frac{\pi}{2}, \lambda)} \mu \right) \]

\[ f_h(\frac{\pi}{2}, \lambda) = \int_{S^2} G(i;z^+, \lambda, 1) dv = (2\pi/\lambda)(1 - e^{-\lambda}) \]
SSDF-SG Product

(a) $\lambda=10000$  
(b) $\lambda=2000$  
(c) $\lambda=50$

Graph: Relative Square Error

- Inner Product
- Vector Product

$log_{10}\lambda$
SSDF-SG product with Compression

(a) Ray-Traced

(b) Uncompressed SSDF

(c) SSDF/PCA 384 Terms

(d) SSDF/PCA 144 Terms

(e) SSDF/PCA 48 Terms

(f) SSDF/PCA 16 Terms
Outline

• Reflectance Representation
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• Visibility Representation
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• Lighting & Rendering
Lighting

• Local light Source
  – Represented by a single SG

• Environment Light
  – SGs for diffuse shading
  – MIPMAP for specular shading
Local Light Source

- Point light

\[ L^*(x, i) = G \left( i, \frac{p - x}{\|p - x\|}, -\frac{\|p - x\|^2 \ln \varepsilon}{r^2}, \frac{s}{\|p - x\|^2} \right) \]

- Directional light

\[ L^*(i) = G \left( i, p, -\frac{\ln \varepsilon}{r^2}, s \right) \]
Environment Light

SGs (<10 lobes)  
[Tsai and Shih 2006]

prefiltered MIPMAP  
[Kautz et al. 2000]
Run-time Rendering

\[ R(o) = k_d R_d + k_s R_s(o) \]

\[ R_d = \int_{S^2} L(i) V(i) \max(0, i \cdot n) \, di \]

\[ R_s(o) = \int_{S^2} L(i) \rho_s(o, i) V(i) \max(0, i \cdot n) \, di \]
Run-time Rendering

- **Environment light**

\[
R_d = \left( C^*(i; n_x) \otimes L^*(i) \right) \cdot V_x^d(i)
\]

\[
R_s(o) = \left( C^*(i; n_x) \otimes \rho_s^*(i; o) \otimes V_x^d(i) \right) \cdot L(i)
\]

\[
C^*(i; n_x) = G(i; n_x, \lambda_c, \mu_c), \quad \lambda_c = 2.133, \quad \mu_c = 1.170
\]

- BRDF Slice in SGs
- Cosine Term in SGs
- Visibility in SSDF
- Prefiltered Environment
Run-time Rendering

- Local point light

\[
R_d = \left( L_x^*(i) \otimes C^*(i; z^+) \right) \cdot V_x^d(i)
\]

\[
R_s(o) = \left( L_x^*(i) \otimes C^*(i; z^+) \otimes \rho_{s,x}^*(i; o) \right) \cdot V_x^d(i)
\]

BRDF Slice in SGs

Cosine Term in SGs

Point light in SGs

Visibility in SSDF
Result

All-Frequency Rendering with Dynamic, Spatially-Varying Reflectance

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\textsuperscript{3}Microsoft Research
<table>
<thead>
<tr>
<th>Scene</th>
<th>BRDF Type</th>
<th>Texture Res.</th>
<th>BRDF Size</th>
<th># Vert.</th>
<th># E.L.</th>
<th>SSDF Samp.</th>
<th>SSDF Comp.</th>
<th>SSDF Size</th>
<th>Env. FPS</th>
<th>Pt. FPS</th>
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<tbody>
<tr>
<td>Teapot: Fig.1a, Fig.11ab</td>
<td>Cook-Torrance, (iso, 1 SG)</td>
<td>1024×1024</td>
<td>7.2MB</td>
<td>17k</td>
<td>8</td>
<td>20 min.</td>
<td>30D (25 min.)</td>
<td>2MB</td>
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<td>Dragon: Fig.1b, Fig.11cd</td>
<td>Ward (iso, 1 SG)</td>
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<td>Dish+Balls: Fig.1c, Fig.12abc</td>
<td>Ashikhmin-Shirley (aniso, 7 SGs)</td>
<td>512×1024</td>
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<td>28.5k</td>
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<td>45 min.</td>
<td>20D (30 min.)</td>
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<td></td>
<td>measured card (iso, 2 SGs)</td>
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<td></td>
<td>measured satin (aniso, 5 SGs)</td>
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<td>22.4MB</td>
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<td></td>
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</tbody>
</table>
Conclusion - Pros

• SG mixtures for microfacet-based reflectance
  – Highly specular
  – Speed up rendering

• Compressed SSDFs
  – Ghost-free
  – Per-pixel interpolation
Conclusion - Cons

• Pre-computed visibility
  – Limited static scenes
  – Maybe solve this problem by combining other methods.

• Consider only direct shadowing effects

• Many SG lobes are required for anisotropic BRDFs.
Thank you!