SIGGRAPH 2005 Course #6:

Advanced Topics on Clothing Simulation and Animation

Organizers: Hyeong-Seok Ko
Kwang-Jin Choi

Lecturers: Robert Bridson
Dongliang Zhang

A Brief History of the Course

- SIGGRAPH 1998
  - "Cloth and Clothing in Computer Graphics"
  - Organized by House
- SIGGRAPH 2003
  - "Clothing Simulation and Animation"
  - Organized by Breen and Ko
- SIGGRAPH 2005
  - "Advanced Topics on Clothing Simulation and Animation"
  - Organized by Choi and Ko

Course Schedule

1st Session

1:45~2:00 Introduction (Ko)

2:00~3:00 Physical Model of Cloth I (Ko)

3:00~3:30 Physical Model of Cloth II (Choi)

2nd Session

3:45~4:30 Collision Handling (Bridson)

4:30~5:00 Cloth Design and Applications (Zhang)

5:00~5:30 Current State-of-the Art / Challenges Ahead (Ko)

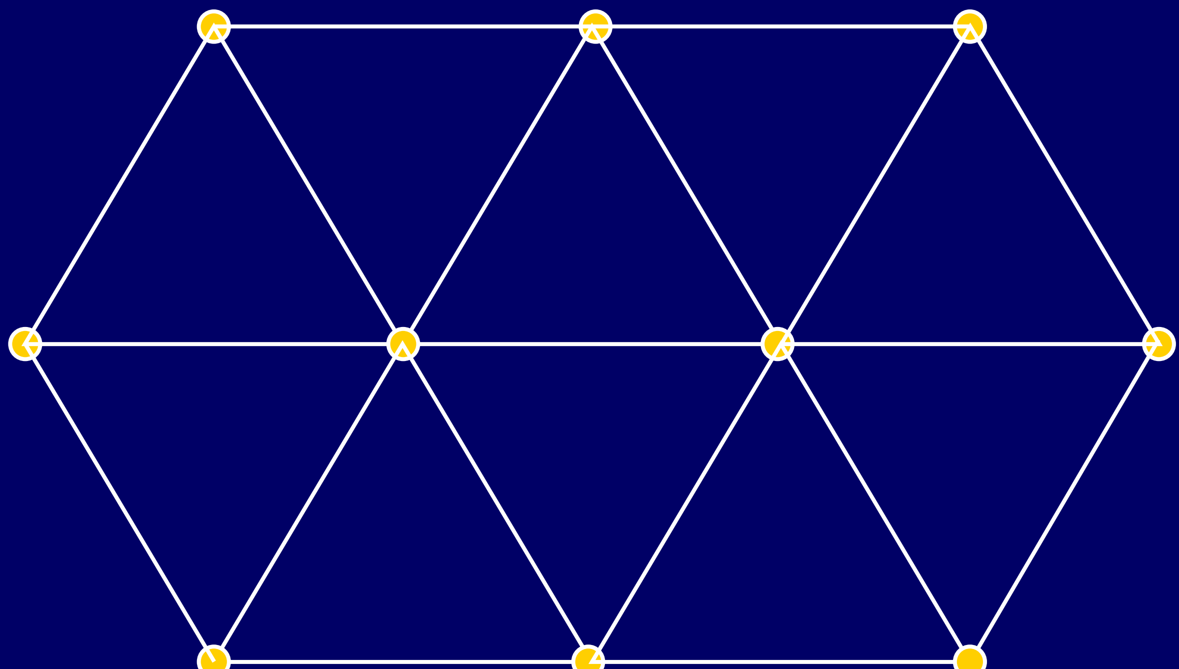
Introduction

How Cloth Simulation Works?

Hyeong-Seok Ko
Seoul National Univ.
Graphics & Media Lab

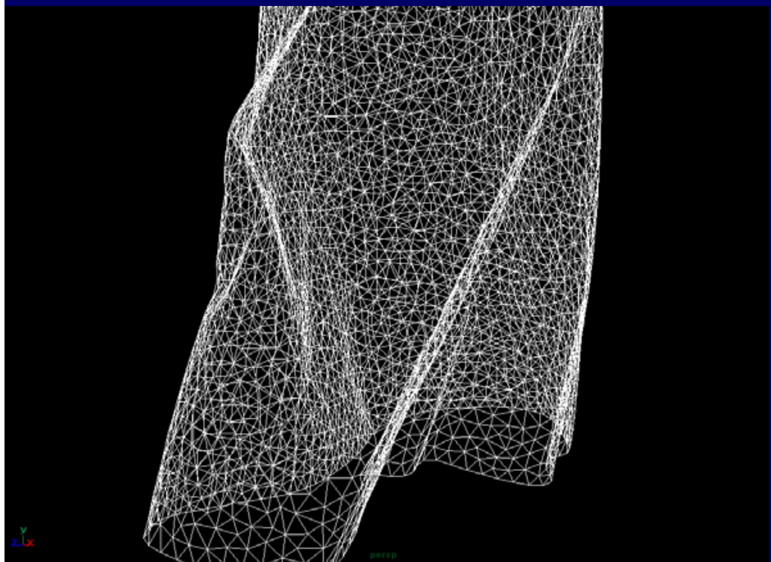
First of All, How to Represent Cloth?

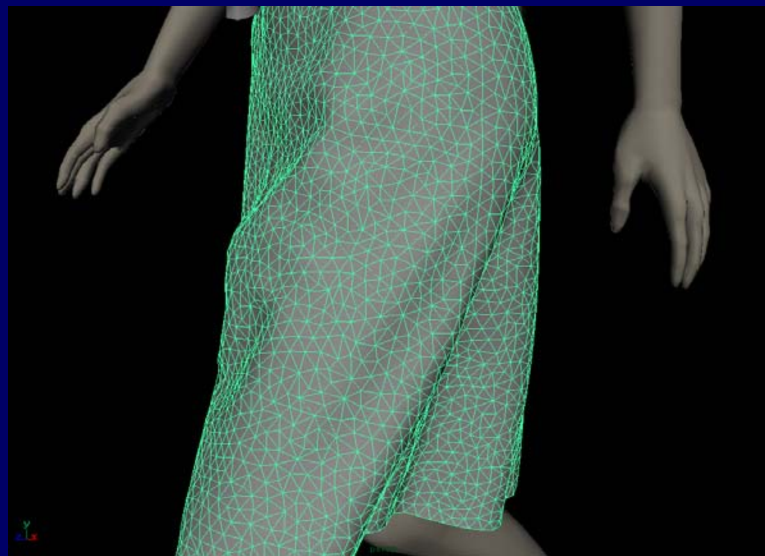
Particles interconnected by springs



Particle-based Cloth Simulation

- Repeat the following:
  1. Find the new position of the particles
  2. Draw the surface from the particles





How Does Cloth Simulation Work?

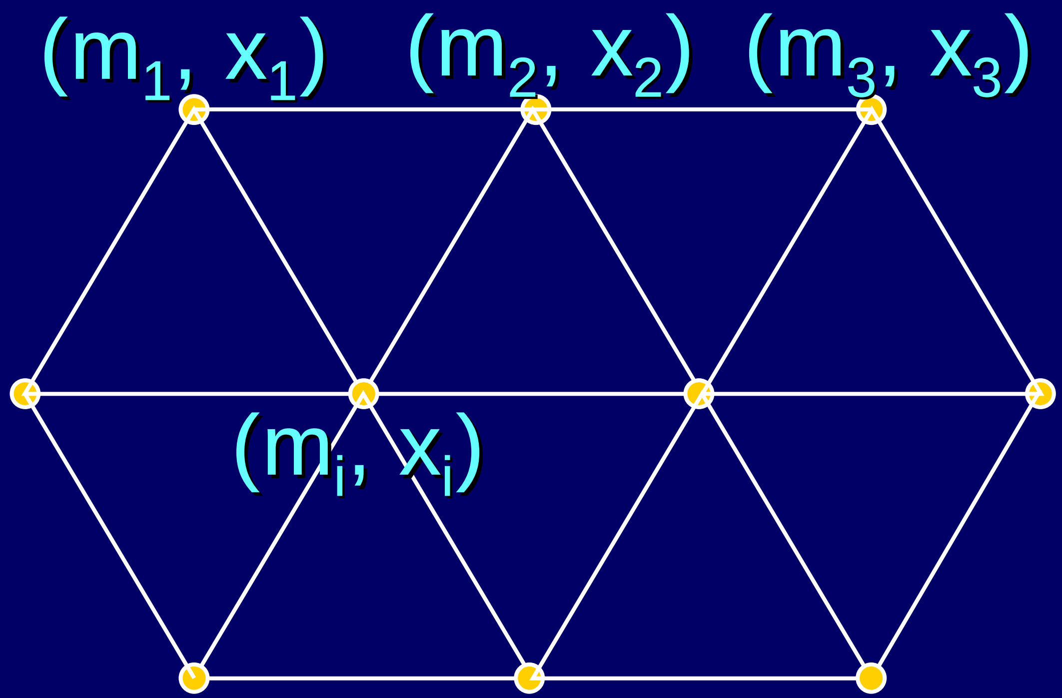
- Finding Governing Equation
- Solving the Equation
- Collision Handling

Finding the Governing Equation

Physically-based Simulation

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$

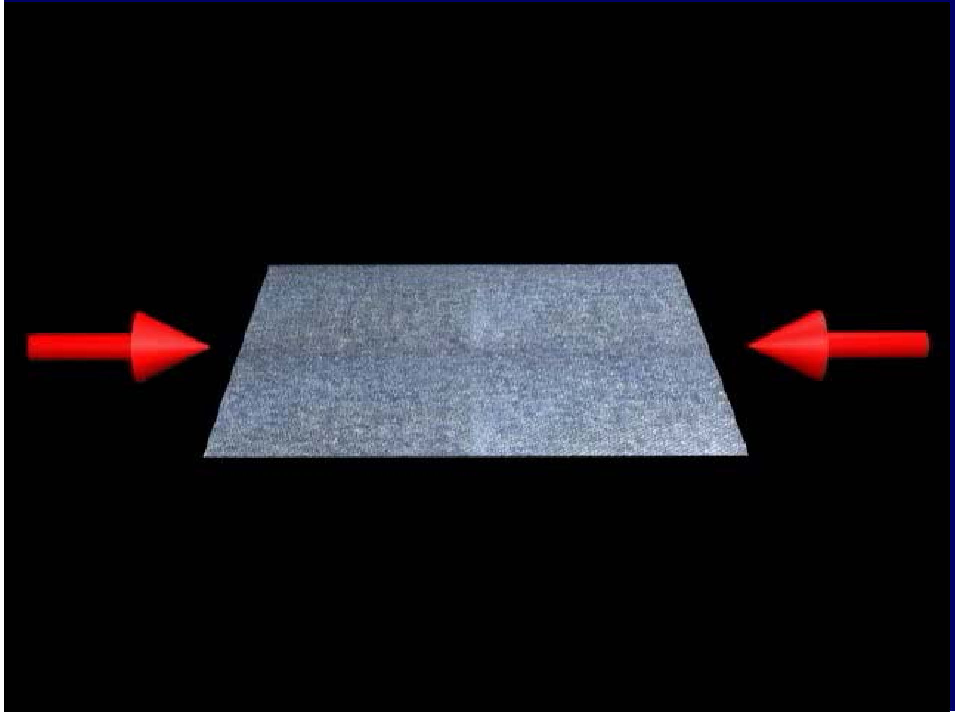
$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

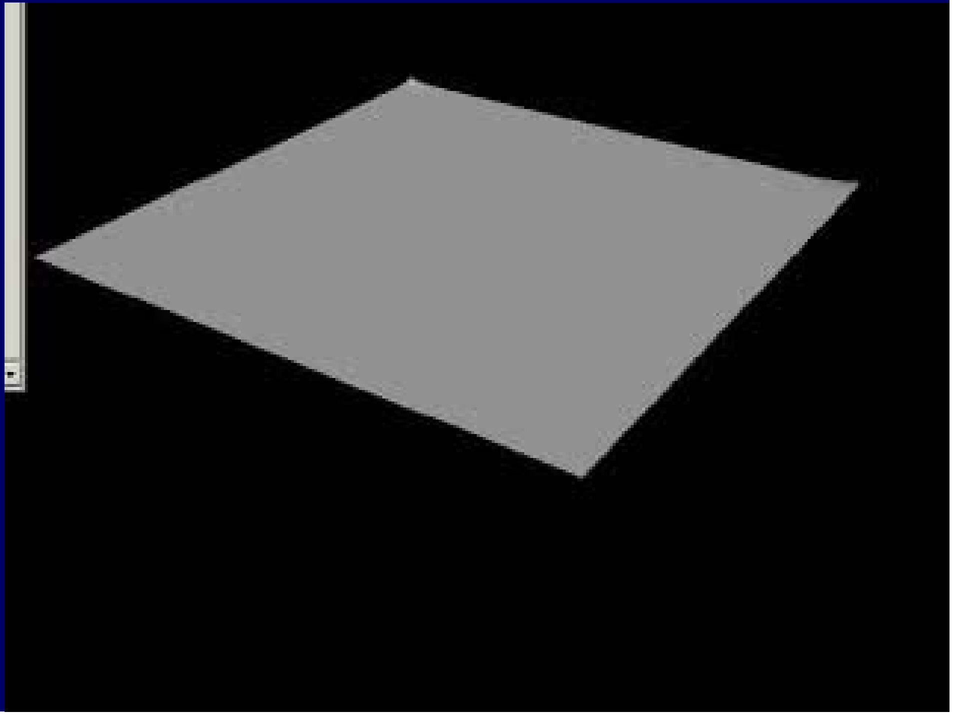


$$M_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix}$$

Physically-based Simulation

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$





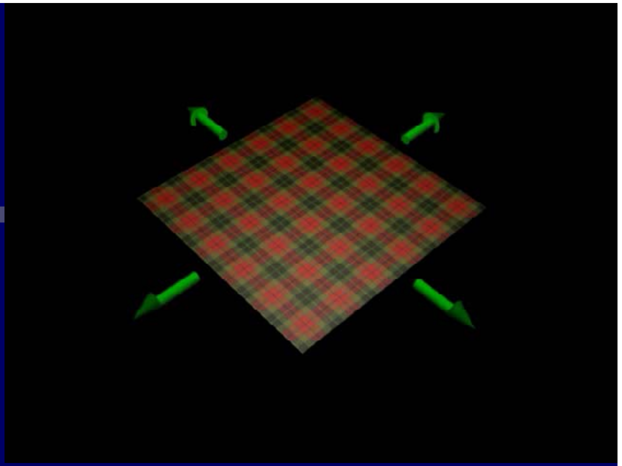
Solving the Equation

Now, numerically solve

$$\ddot{x}(t) = M^{-1}(-\frac{\partial E}{\partial x} + F)(t)$$

```
while(1) {
  compute the force part at t;
  compute accel, vel, pos;
  t = t + Δt;
}
```

We want to use large Δt



- Animators love large Δt
- Large Δt can cause inaccuracy
- Large Δt can cause a more serious problem
- We must have a stable algorithm

Course Schedule

Finding the Governing Equation
Solving the Equation

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5:00~5:30 Current State-of-the Art / Challenges Ahead (Ko)

Collision Handling

Collision Handling

- Consists of Detection/Resolution
- Undetected collision can cause a lot of trouble.
- Cloth simulation is abundant of challenging cases for collision detection.
  - A difficult problem: Multiple simultaneous collisions.
  - Try this: simulate a person wearing multiple pieces of clothes.

"Collision Handling" by Bridson (Session 2)

Additional Parts

1st Session

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5:00~5:30 Current State-of-the Art / Challenges Ahead (Ko)

Physical Model of Cloth I

Hyeong-Seok Ko
Seoul National Univ.
Graphics & Media Lab

Physical Model of Cloth I

1. Understanding the problem
2. Immediate buckling model (IBM)
3. Stability analysis of the IBM
4. Damping analysis in clothing simulation

Understanding the Problem

What are the nontrivial parts in...

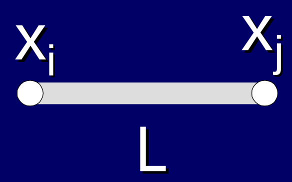
$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$

E

What would you use for E?

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right) \equiv M^{-1}\mathbf{f}(x, \dot{x})$$
$$\equiv M^{-1}\mathbf{f}(x, v)$$

- Cloth is subject to stretch, shear, bending
- $E = E_{stretch} + E_{shear} + E_{bending}$
- What are $E_{stretch}$, E_{shear}, $E_{bending}$?
  - They should be monotonic
  - For example, $E_{stretch} = \{||x_i - x_j|| - L\}^2$



E should accurately represent the property of the cloth

E should not cause problems in numerical integration

Integration of

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right) = M^{-1}\mathbf{f}(x, v)$$

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} v \\ M^{-1}\mathbf{f}(x, v) \end{bmatrix}$$

$$\begin{bmatrix} \Delta x \\ \Delta v \end{bmatrix} = \Delta t \begin{bmatrix} v^0 \\ M^{-1}\mathbf{f}(x^0, v^0) \end{bmatrix} \equiv \Delta t \begin{bmatrix} v^0 \\ M^{-1}\mathbf{f}^0 \end{bmatrix}$$

Explicit Integration

We want Implicit Integration

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right) = M^{-1}\mathbf{f}(x, v)$$

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Explicit Integration

Baraff and Witkin, *Large Steps in Cloth Simulation*, SIGGRAPH '98

We want Implicit Integration

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right) = M^{-1}\mathbf{f}(x, v)$$

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} v \\ M^{-1}\mathbf{f}(x, v) \end{bmatrix}$$

$$\begin{bmatrix} \Delta x \\ \Delta v \end{bmatrix} = \Delta t \begin{bmatrix} v^0 + \Delta v \\ M^{-1}\mathbf{f}(x^0 + \Delta x, v^0 + \Delta v) \end{bmatrix}$$

$$\left(\mathbf{I} - \Delta t \mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} = \Delta t \mathbf{M}^{-1}\left(\mathbf{f}^0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}^0\right)$$

HW 1: Derive this

Into a Schematic Equation

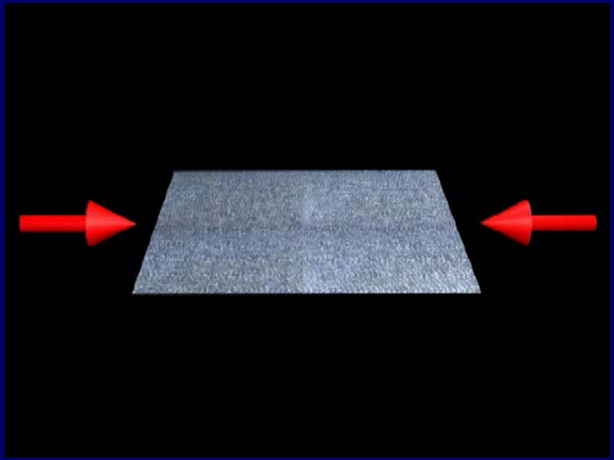
$$\left(\frac{\mathbf{M}}{\Delta t^2} - \frac{\partial \mathbf{f}}{\Delta t \partial \mathbf{v}} - \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} \Delta t = \left(\mathbf{f}^0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}^0\right)$$

$$\left(\mathbf{M} - \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} = \Delta t \left(\mathbf{f}^0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}^0\right)$$

$$\left(\mathbf{I} - \Delta t \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} = \Delta t \mathbf{M}^{-1} \left(\mathbf{f}^0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}^0\right)$$

$$\left(\frac{\mathbf{M}}{\Delta t^2} + \mathbf{K}\right)\Delta \mathbf{x}^n = \mathbf{f}^{n-1}$$

E can cause problems in Num. Int.



$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$

$$\left(\mathbf{I} - \Delta t \mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} = \Delta t \mathbf{M}^{-1}\left(\mathbf{f}^0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}^0\right)$$

$$\left(\frac{\mathbf{M}}{\Delta t^2} + \mathbf{K}\right)\Delta \mathbf{x}^n = \mathbf{f}^{n-1}$$

Ill-conditioned or indefinite system matrix ➔ Divergence!

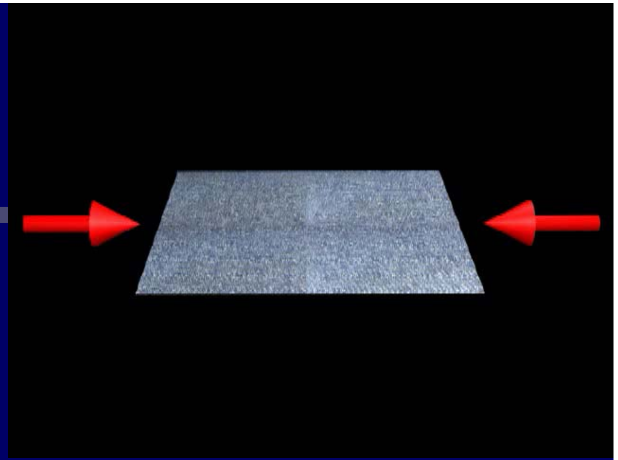
Quest for a New Physical Model

Find a Physical Model

- that makes K always positive definite
- that produces realistic movements of cloth

$$\left(\frac{\mathbf{M}}{\Delta t^2} + \mathbf{K} \right) \Delta \mathbf{x}^n = \mathbf{f}^{n-1}$$

Source of the Problem

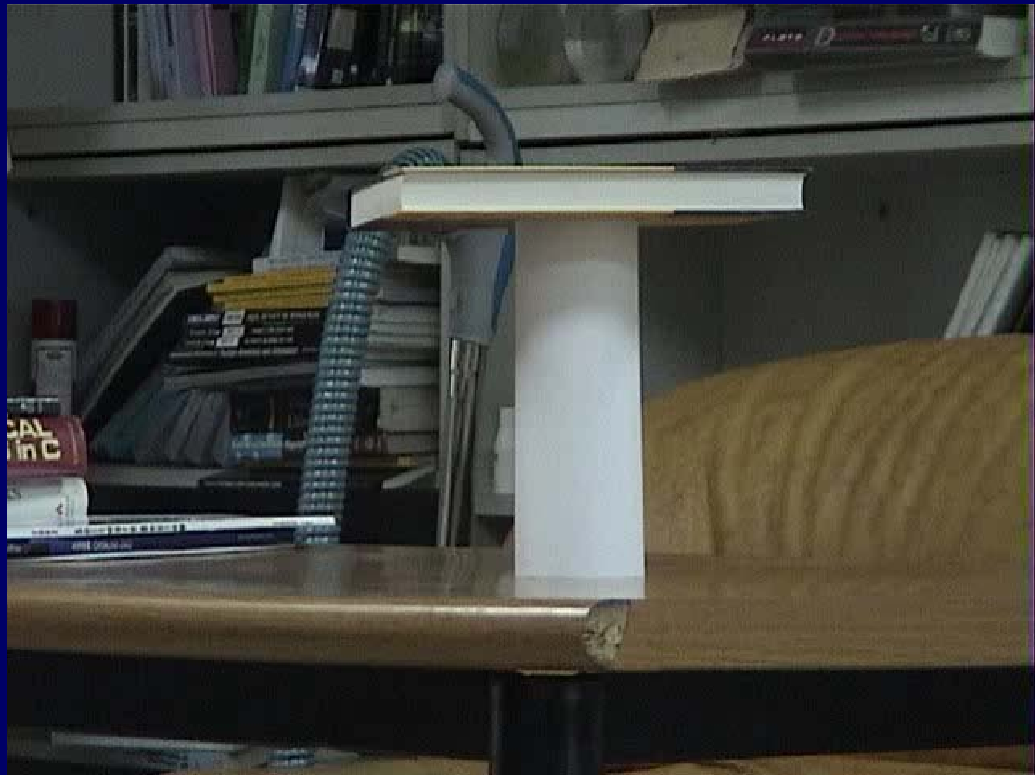


In simulating hookean model,
(compress. stiffness ≈ tens. stiffness)

$$\sigma = E\varepsilon, \quad f = k(x-l)$$

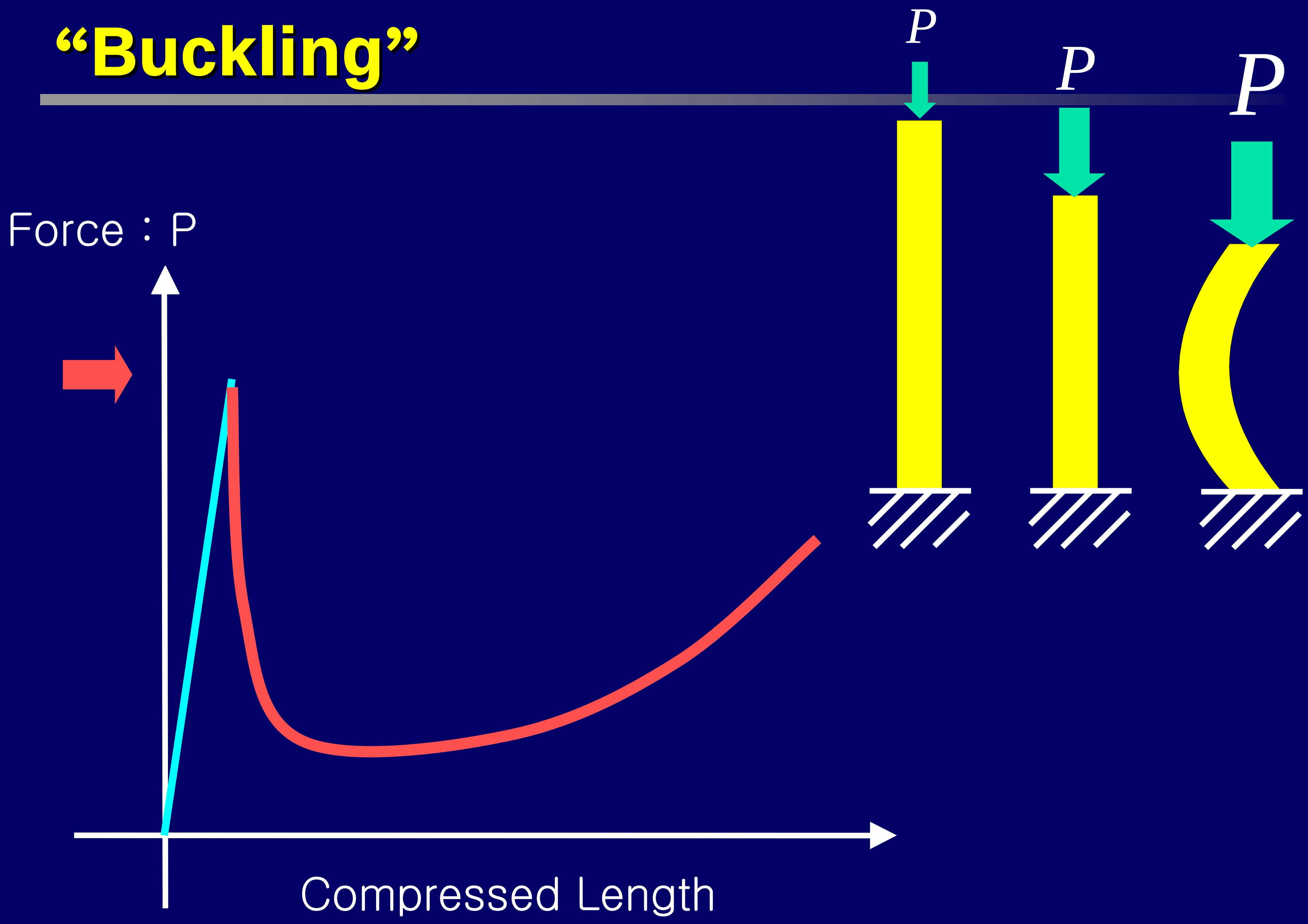
- Instability occurs when cloth is bended or compressed rather than when it is stretched
  - It appears that instability occurs when wrinkles are formed

Compression / Bending, are they Unstable Phenomena?

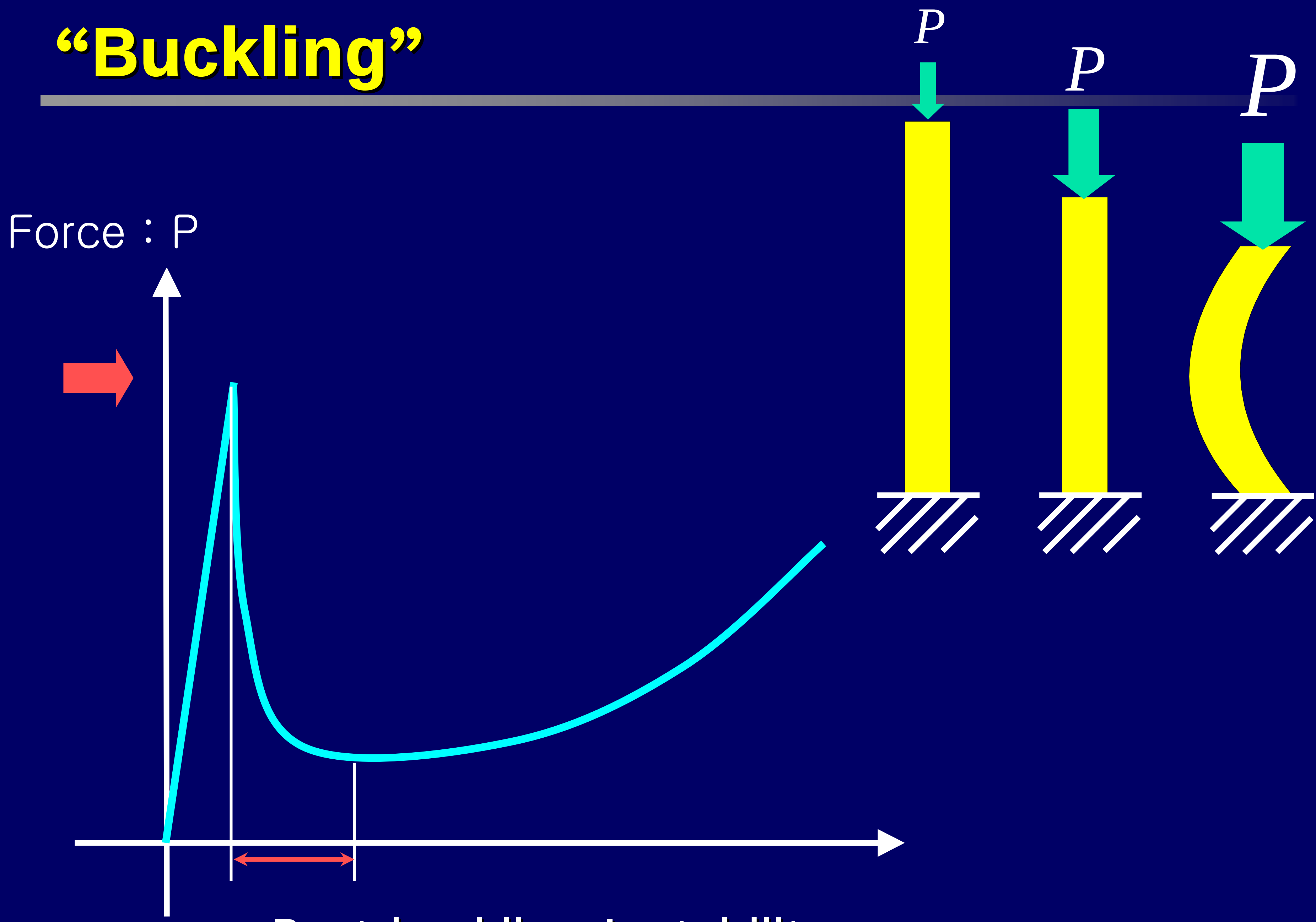


Yes!

"Buckling"



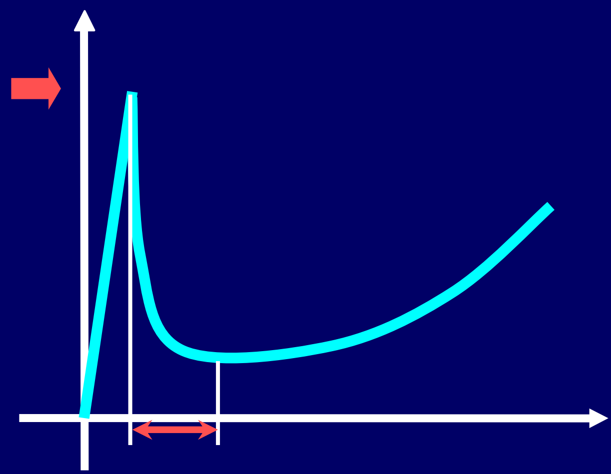
“Buckling”



Post-buckling Instability

Compression / Bending, are they Unstable Phenomena?

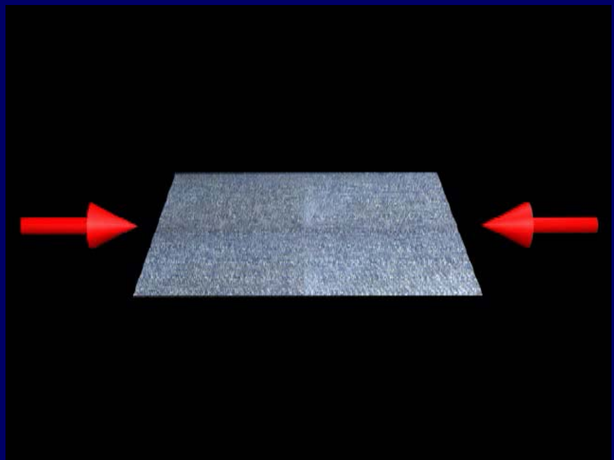
Apparently, No

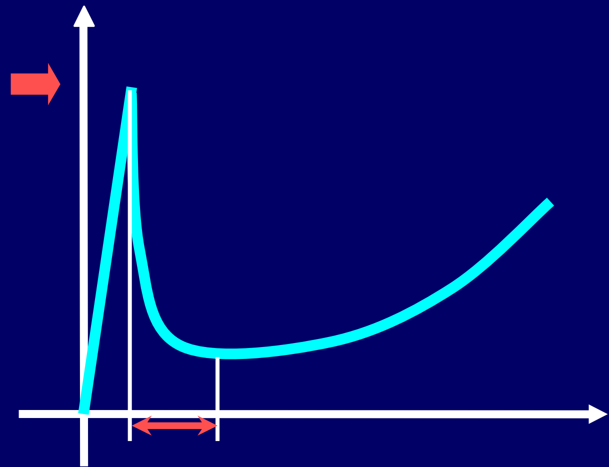


Post-buckling instability passes instantly, and reaches a static equilibrium.

Why was Compression/Bending Unstable in Clothing Simulation?

- Hookean model causes the system matrix indefinite when compression/bending occur



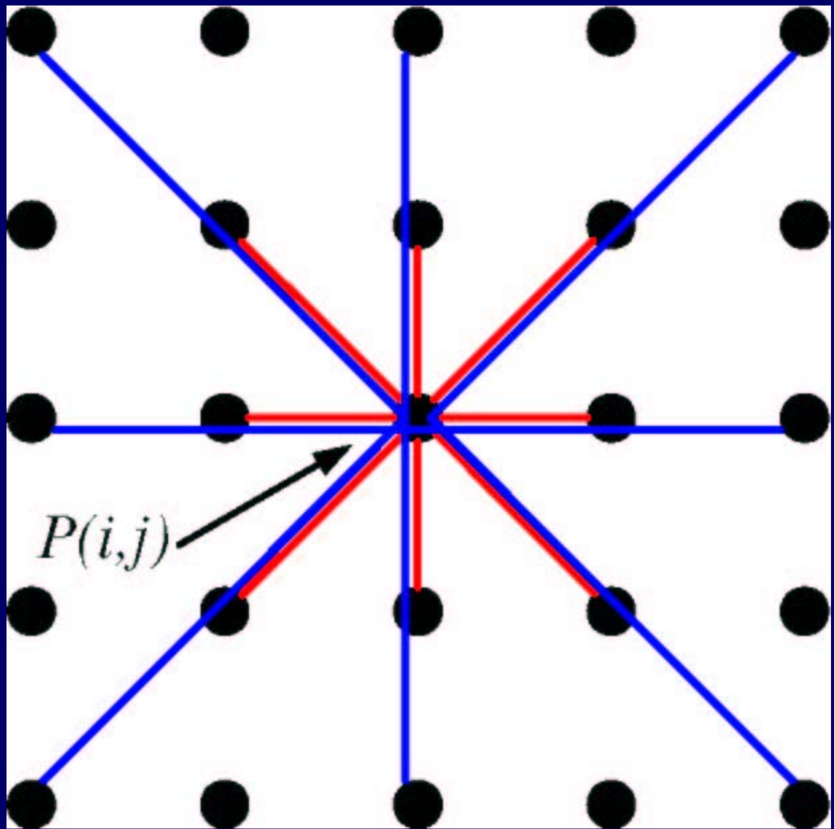


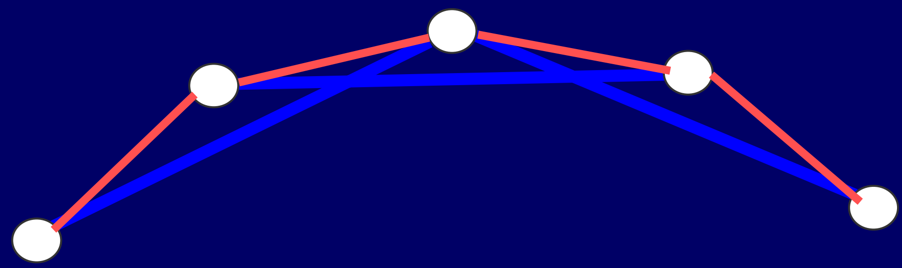
$$\left(\mathbf{I} - \Delta t \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right) \Delta \mathbf{v} = \Delta t \mathbf{M}^{-1} \left(\mathbf{f_0} + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}_0 \right)$$

Suggestion for a Stable Solution

- Hookean model causes the system matrix indefinite when compression/bending occur
- The model works well for stretch
- Suggested solution: use different physical models for stretch and compression
  - Type 1 for stretching
  - Type 2 for compression/bending

How to Connect Springs?





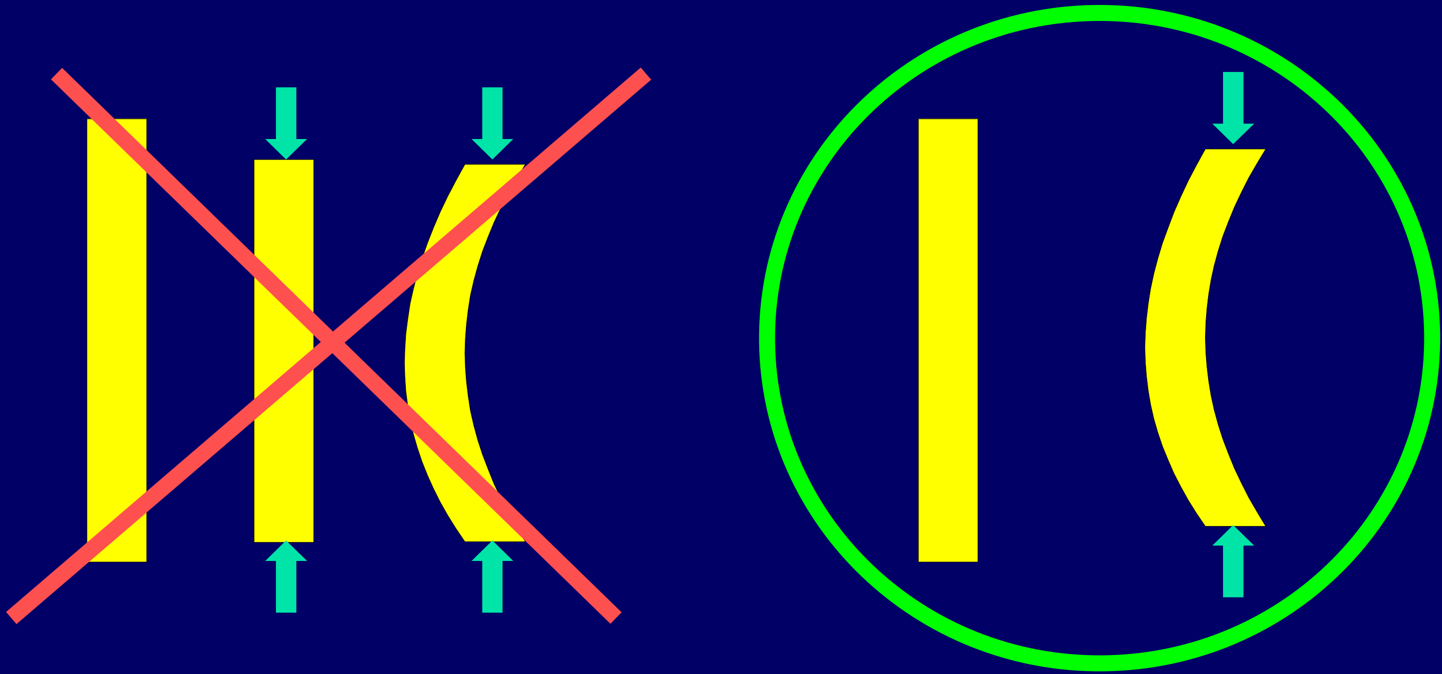
- Red Links = Type 1 (for stretch)
- Blue Links =Type 2 (for compression and bend)

Immediate Buckling Model

How to realize...

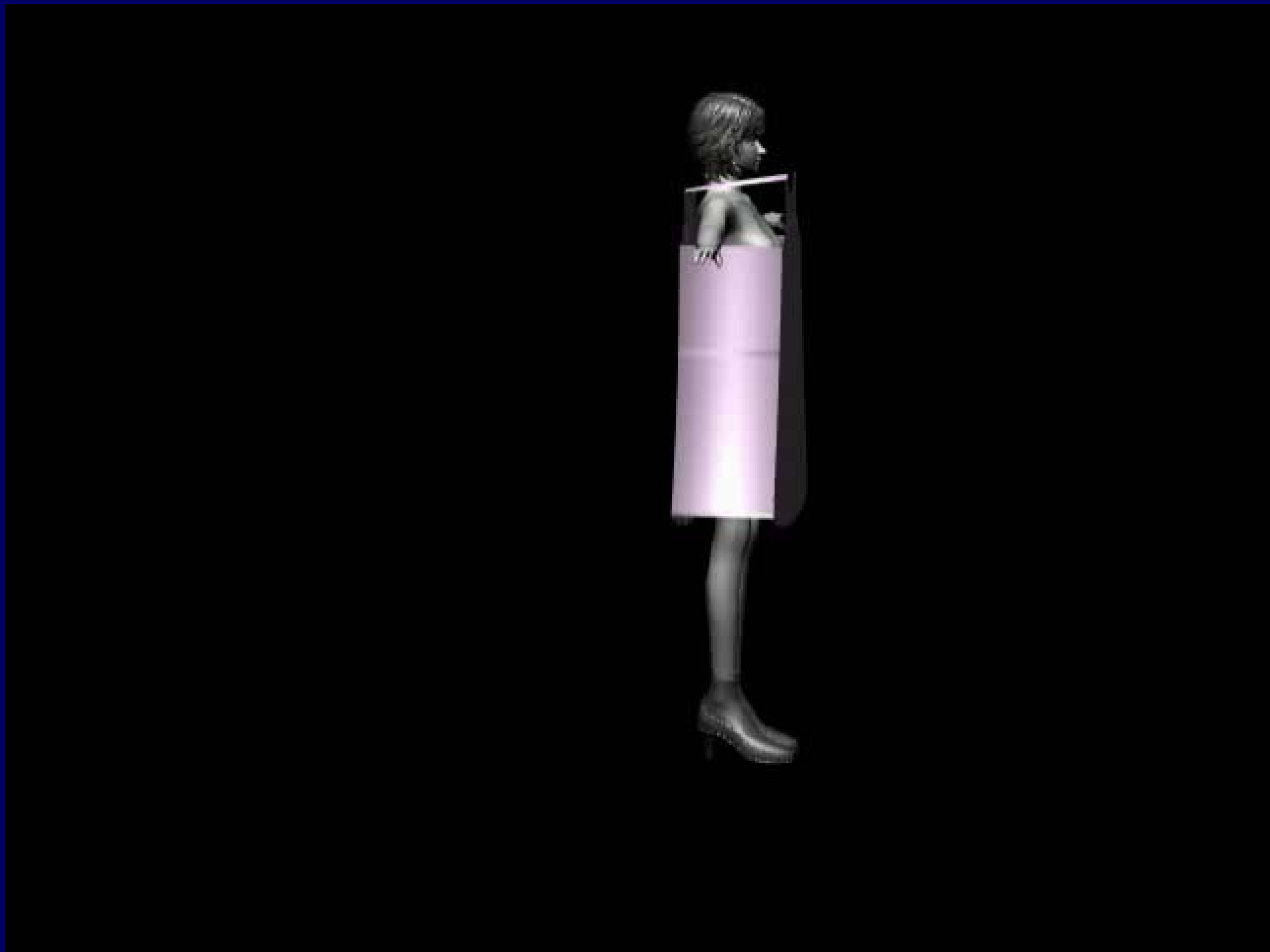
Post-buckling instability passes instantly, and reaches a static equilibrium.

Compression causes <u>bending</u> rather than <u>shortening</u>



Immediate Buckling Assumption

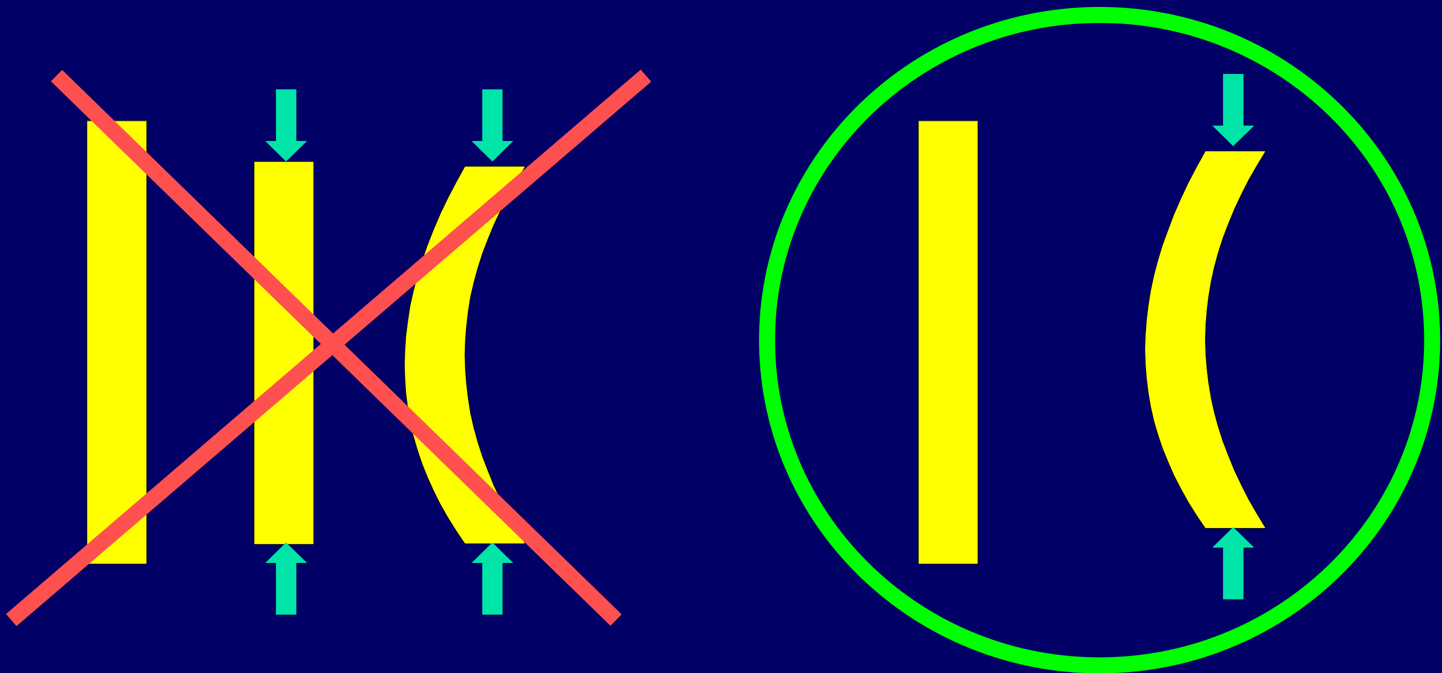
Experiment



# of particles = 5608, Δt=0.011s, CPU=0.51s

How to implement this idea?

Compression causes <u>bending</u> rather than <u>shortening</u>

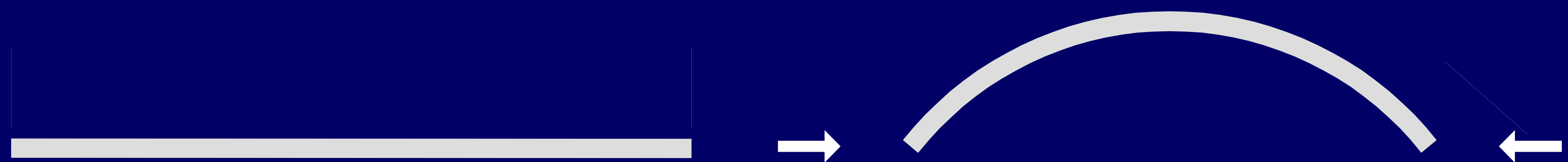


Immediate Buckling Assumption

The Steps We Took

1. Predict the shape at the static equilibrium after buckling
2. Formulate the energy function from the deformed shape
3. Derive the force, Jacobian, etc.

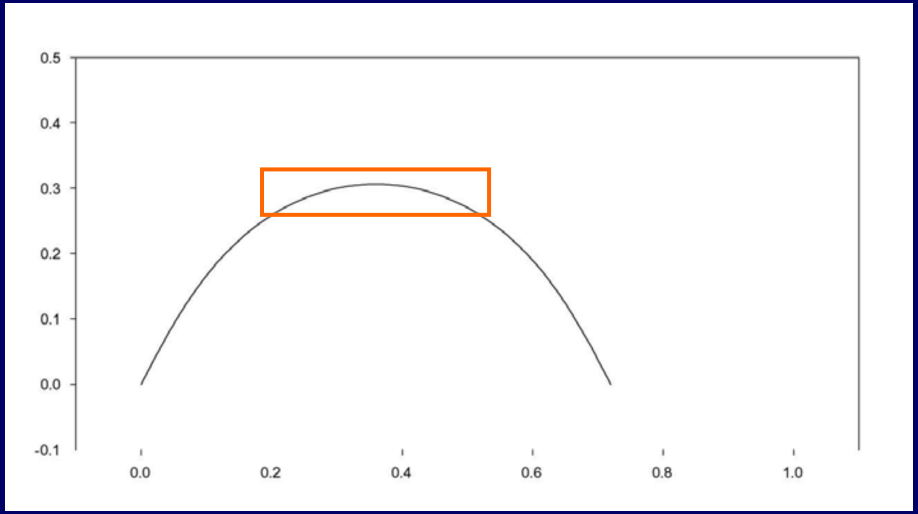
Predicting the Static Equilibrium



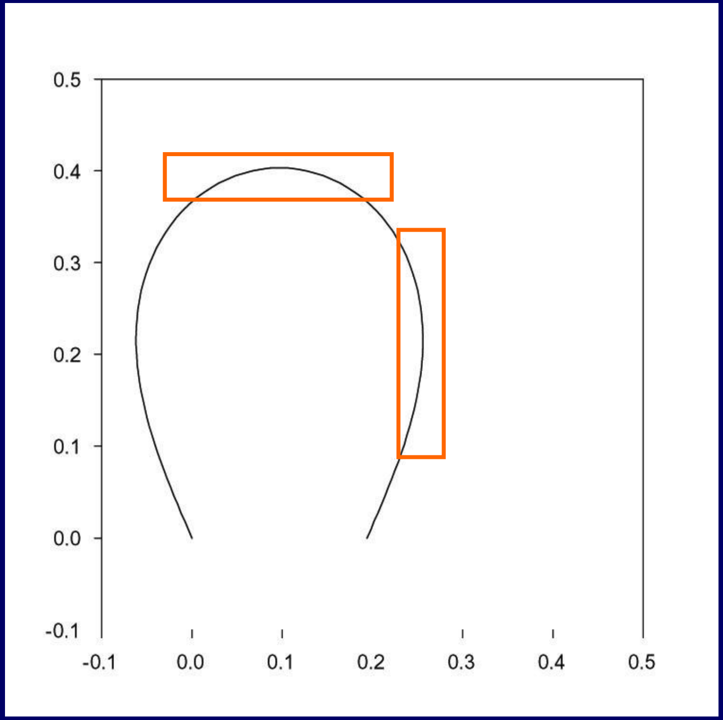
Predicting the Static Equilibrium

Momentum Equilibrium Equation

$$k_b \kappa + Py = 0$$



$P/k_b = 23.5$

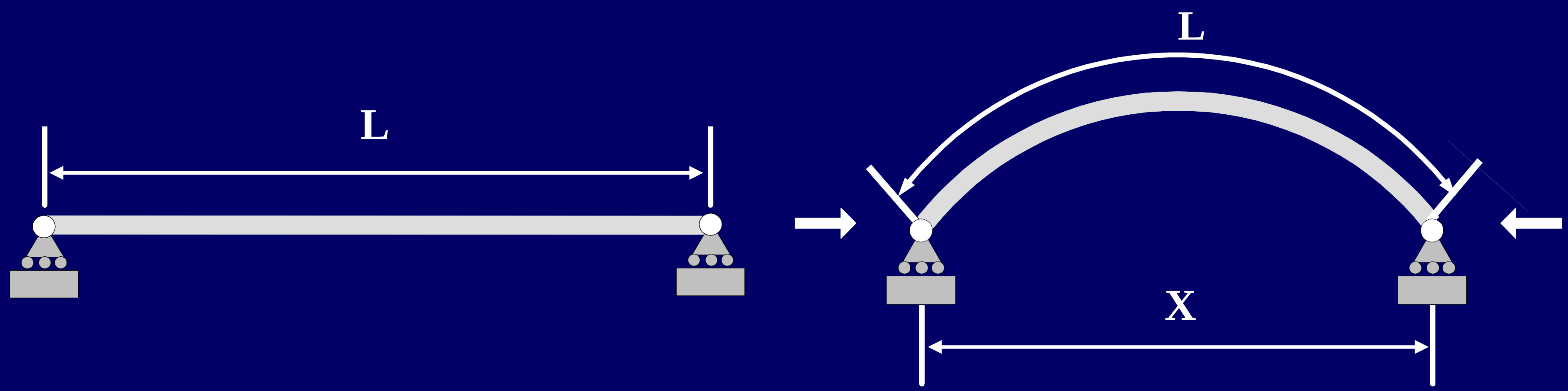


$P/k_b = 35.25$

Equilibrium shape is similar to a circular arc.

Simplifying Assumptions

- Equilibrium shape = circular arc
- Arc length remains constant



Equilibrium shape is completely characterized when x_i and x_j are given.

Energy Function in terms of $x_{ij}=x_j-x_i$

- The energy function $$E=\frac{1}{2}\int_0^L M\kappa dx$$
- Since $M \propto \kappa$ & curvature is constant $$E=\frac{1}{2}k_b L\kappa^2$$
- Since arc length is constant $$\kappa=\frac{2}{L}\text{sinc}^{-1}\left(\frac{|x_{ij}|}{L}\right)$$

$$E=\frac{1}{2}k_b L\left[\frac{2}{L}\text{sinc}^{-1}\left(\frac{|x_{ij}|}{L}\right)\right]^2$$

The Steps We Took

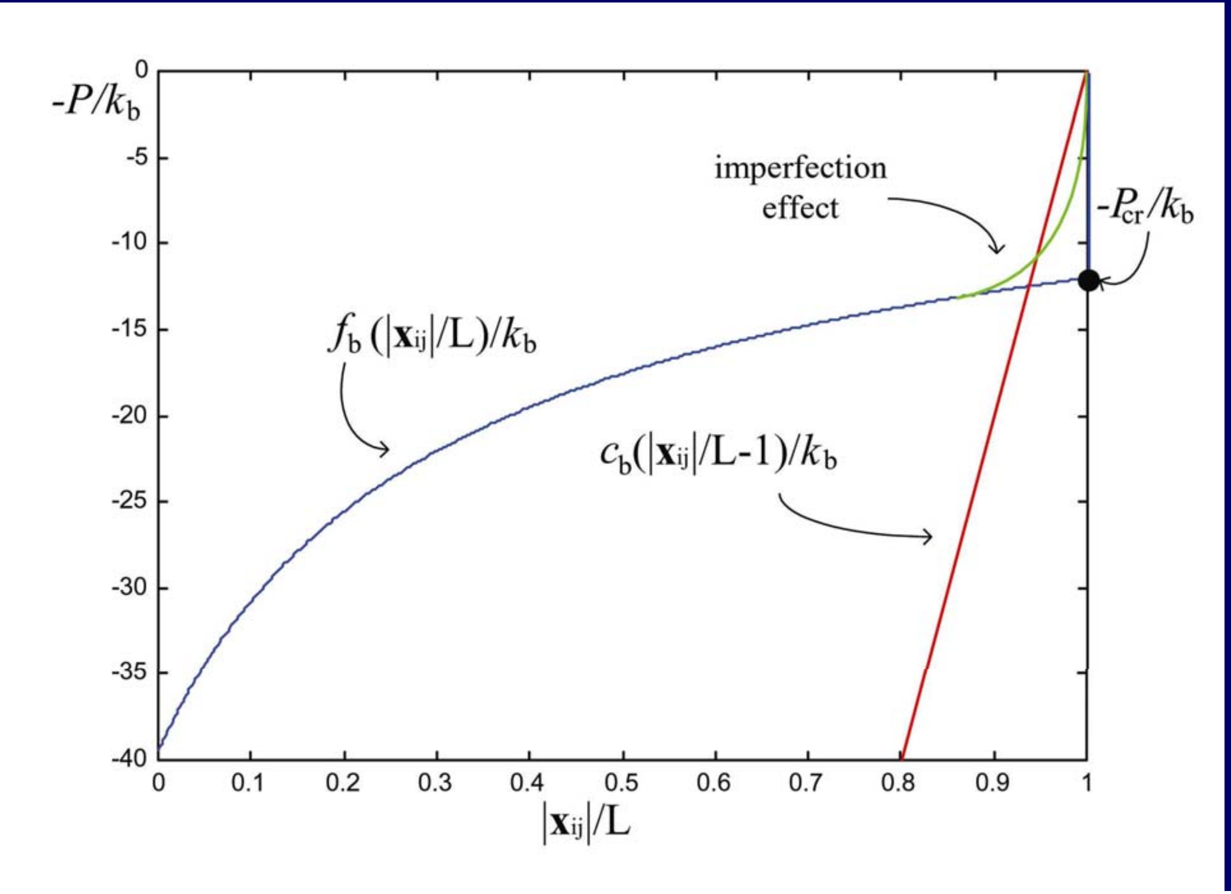
1. Predict the shape at the static equilibrium after buckling
2. Formulate the energy function from the deformed shape
3. Derive the force, Jacobian, etc.

$$\left(\mathbf{I} - \Delta t \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right) \Delta \mathbf{v} = \Delta t \mathbf{M}^{-1} \left(\mathbf{f}^0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}^0 \right)$$

Force Derivation

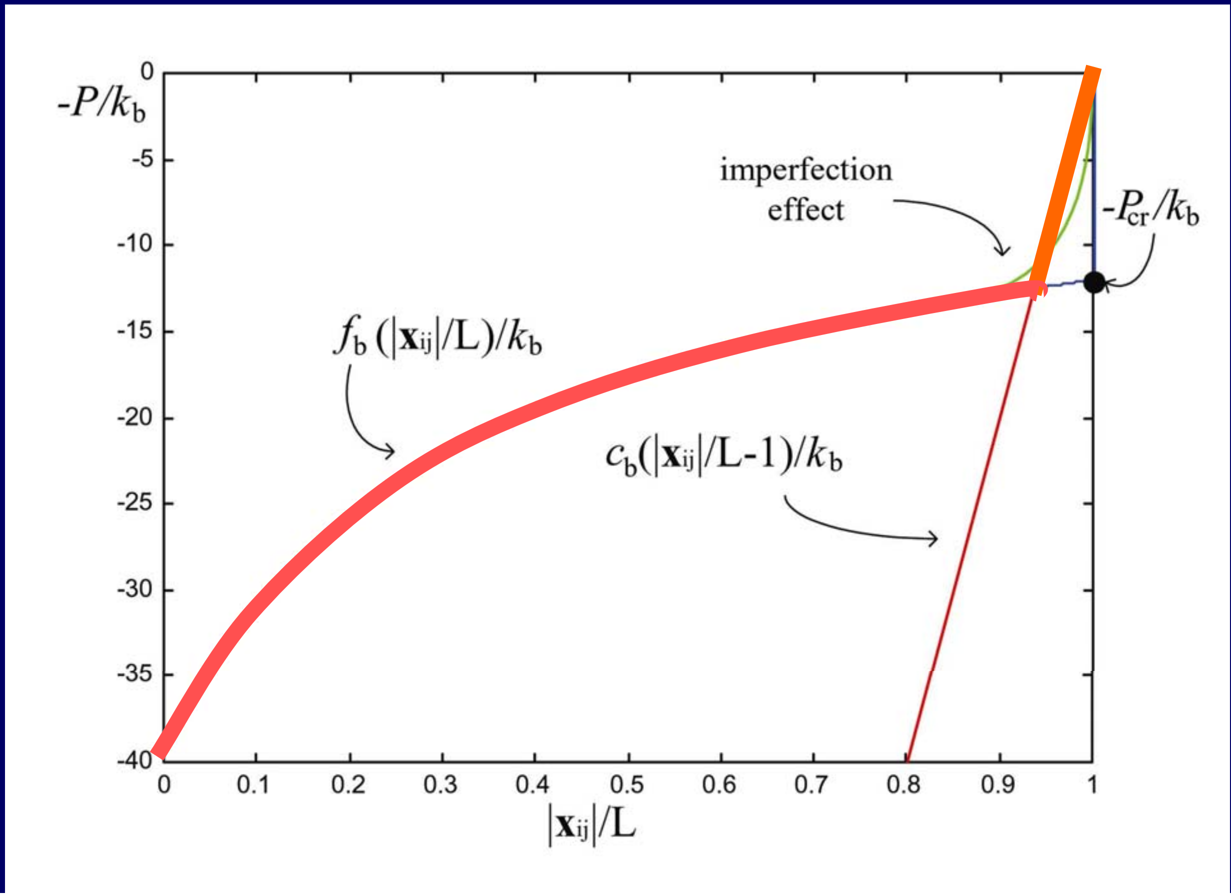
$$E = \frac{1}{2} k_b L \left[\frac{2}{L} \mathrm{sinc}^{-1}\left(\frac{|x_{ij}|}{L} \right) \right]^2$$

$$\mathbf{f}_i = -\frac{\partial E}{\partial \mathbf{x}_i} = \sum_{j \in N(i)} f_b(|x_{ij}|) \frac{x_{ij}}{|x_{ij}|},$$

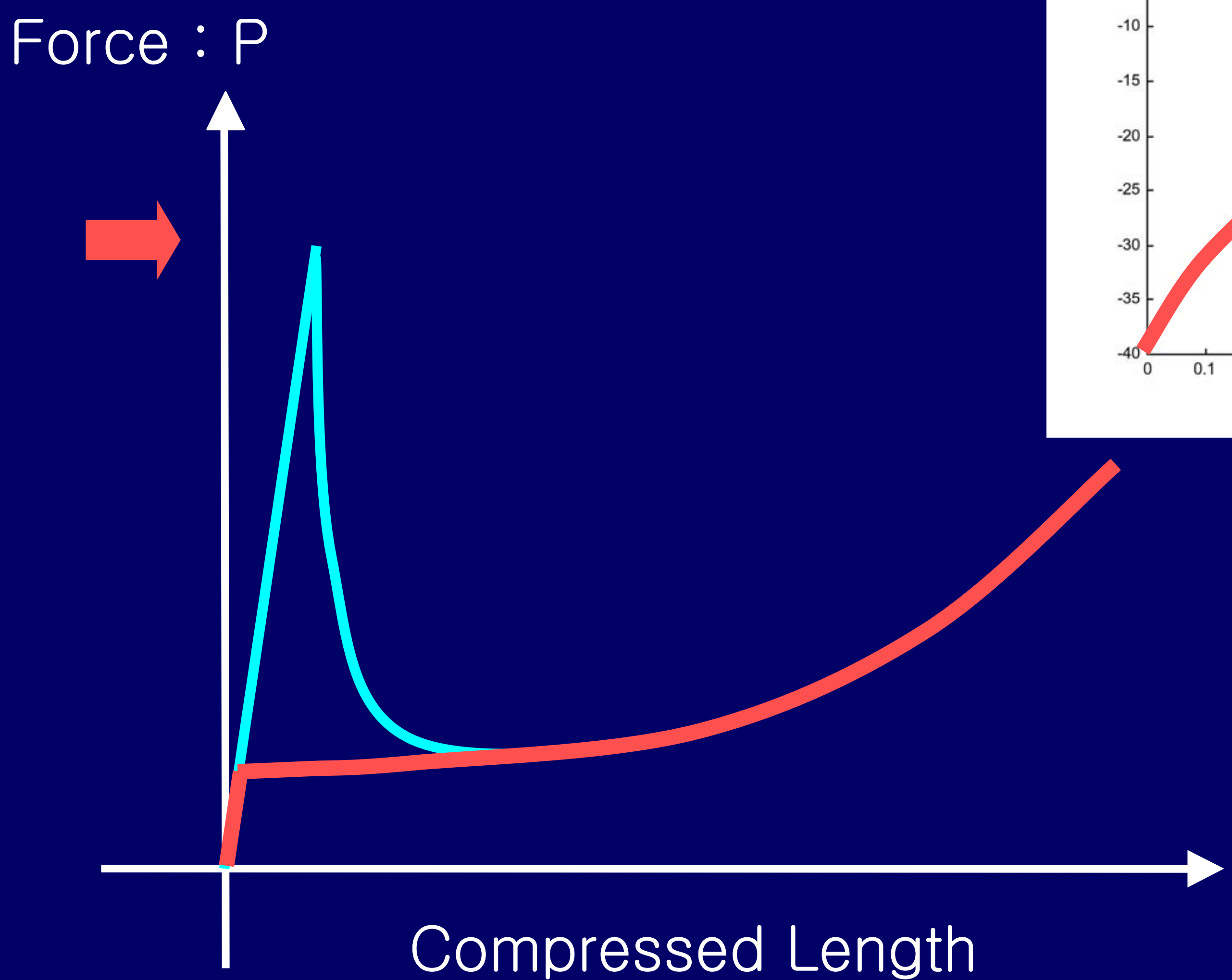


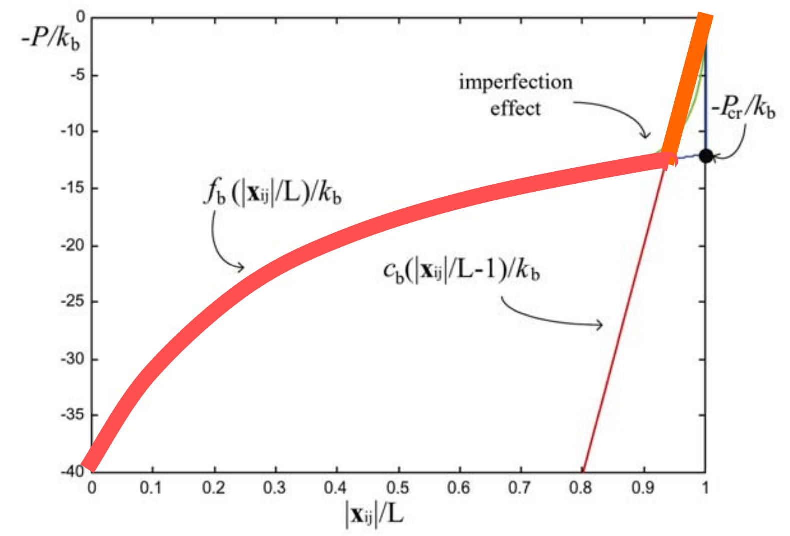
The Force-Deflection Curve Finally Used

$$f_b^* = \begin{cases} c_b\left(\left|x_{ij}\right| - L\right) : f_b < c_b\left(\left|x_{ij}\right| - L\right) \\ f_b : \text{otherwise} \end{cases}$$



New Force-Displacement Curve





Meaning of the Force Jacobian $\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}$

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{\partial \sum_{k \in N} \mathbf{f}_{ik}}{\partial \mathbf{x}_j} = \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{x}_j}$$

HW 2: Why?

The rate of change of f_i as x_j changes

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_i} = -\sum_{j \neq i} \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{x}_j}$$

HW 3: Why?

The rate of change of f_i as x_i changes

Stiffness!

Derivation of Force Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{d f_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} + \frac{f_b^*}{|x_{ij}|} \left(I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}\right)$$

HW 4: Derive this

We can Start Implicit Integration

$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_{\mathbf{0}} + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0 \right)$$

For now, forget about the damping term.

$$\left(\mathbf{I} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_{\mathbf{0}} + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0 \right)$$

Stability Analysis of Immediate Buckling Model

Property of the Force Jacobian $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$

$$\left(\mathbf{I} - \Delta t^2 \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right) \Delta \mathbf{v} = \Delta t \mathbf{M}^{-1} \left(\mathbf{f_0} + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}_0\right)$$

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{d f_b^*}{d\left|x_{ij}\right|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} + \frac{f_b^*}{\left|x_{ij}\right|} \left(I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}\right)$$

- What is the property of this 3×3 matrix?
- What is the property of the global 3n×3n matrix?

The Properties of $A = \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}$

$$\det\left(\frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} - \lambda I\right) = \lambda^2 \left(1 - \lambda\right)$$

HW 5: Prove it

$$\therefore \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} \text{ is positive definite.}$$

$\frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} z$ produces the projection of z along x_{ij}

HW 6: Verify this

The Properties of $I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} = I - A$

$$\left(I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} \right) \text{is positive definite.}$$

HW 7: Why?

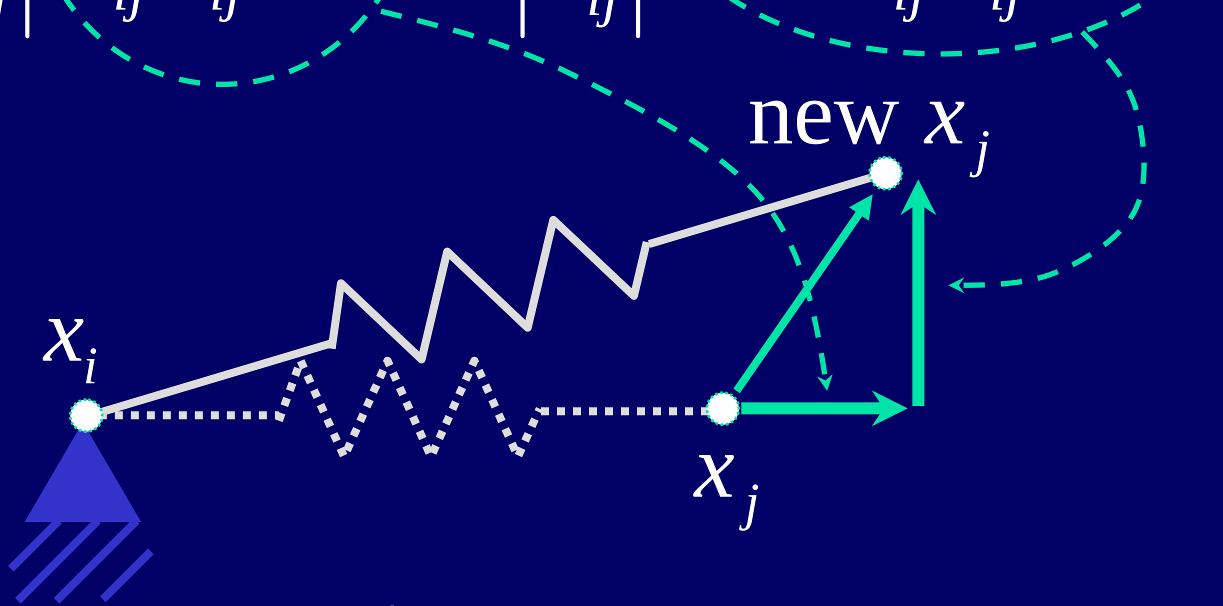
$$\left(I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} \right) z \ \text{ produces the projection of } z$$

on a plane orthogonal to x_{ij}

HW 8: Verify this

They Work as Projection Operators

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} d\mathbf{x}_j = \frac{df_b^*}{d\left|x_{ij}\right|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} d\mathbf{x}_j + \frac{f_b^*}{\left|x_{ij}\right|} \left(I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}\right) d\mathbf{x}_j$$



$\frac{df_b^*}{d\left|x_{ij}\right|}$ = in-plane stiffness

$\frac{f_b^*}{\left|x_{ij}\right|}$ = out-of-pl stiffness

Property of the Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d\left|x_{ij}\right|}\frac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}} + \frac{f_b^*}{\left|x_{ij}\right|}\left(I - \frac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}}\right)$$

- $\frac{df_b^*}{d\left|x_{ij}\right|}$ is positive.
- But $\frac{f_b^*}{\left|x_{ij}\right|}$ is negative.
- So the sum is not definite
- If 2nd term is omitted, it becomes pos. def.

Property of the Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d\left|x_{ij}\right|}\frac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}}$$

$$\left(\mathbf{I} - \Delta t^2 \mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} = \Delta t \mathbf{M}^{-1}\left(\mathbf{f}_0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0\right)$$

- $\frac{df_b^*}{d\left|x_{ij}\right|}$ is positive.
- But $\frac{f_b^*}{\left|x_{ij}\right|}$ is negative.
- So the sum is not definite
- If 2nd term is omitted, it becomes pos. def.
- What is the effect of it?
- Is the global matrix negative definite?

Is the Global Matrix Neg. Def.?

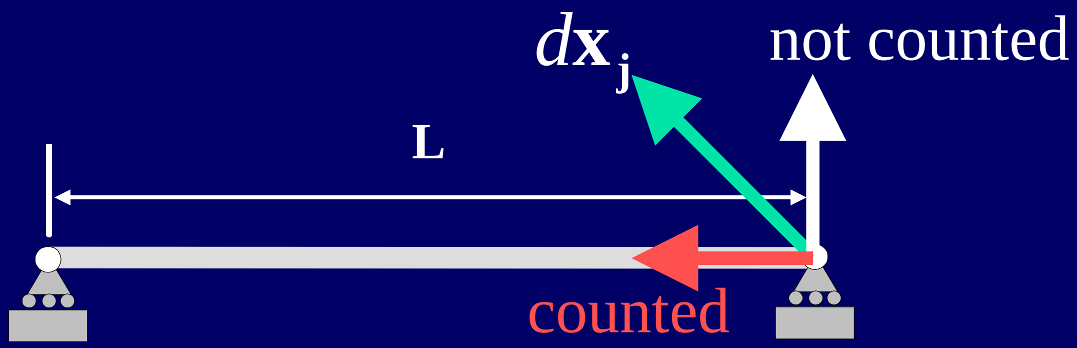
$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1} & \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_n} \\ \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_1} & \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_1} & \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_n} \end{bmatrix} = \begin{bmatrix} -\sum_{j \neq 1} \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_j} & \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_n} \\ \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_1} & -\sum_{j \neq 2} \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_j} & \cdots & \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_1} & \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_2} & \cdots & -\sum_{j \neq n} \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_j} \end{bmatrix}$$

Yes

HW 9: Prove it.

What is the meaning of Omitting the Second Term?

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}$$



- Does it mean ignoring the orthogonal component?
- No.
  - Remember, ∂f/∂x was introduced for implicit integration

In Global Implicit Integration...

$$\left(\mathbf{I} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta\mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_0 + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0\right)$$

Looking at the i-th row

$$LHS = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -h^2\mathbf{M}_i^{-1}\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_{j1}} & \cdots & \mathbf{I} - h^2\mathbf{M}_i^{-1}\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_i} & \cdots & -h^2\mathbf{M}_i^{-1}\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_{jN}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \Delta\mathbf{v}_1 \\ \Delta\mathbf{v}_2 \\ \vdots \\ \Delta\mathbf{v}_n \end{bmatrix}$$

$$= \left(\mathbf{I} + \sum_{j \neq i} h^2\mathbf{M}_i^{-1}\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}\right)\Delta\mathbf{v}_i - \sum_{j \neq i} h^2\mathbf{M}_i^{-1}\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}\Delta\mathbf{v}_j$$

$$RHS = h\mathbf{M}_i^{-1}\left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}\mathbf{v}_{0j}\right)$$

Comparison with Explicit Method

$$\Delta v_i = hM_i^{-1}\mathbf{f}_i^0$$ Explicit

$$\left(\mathbf{I} + \sum_{j\neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}\right)\Delta \mathbf{v}_i - \sum_{j\neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}\Delta \mathbf{v}_j = h\mathbf{M}_i^{-1}\left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}\mathbf{v}_{0j}\right)$$

Implicit

- In LHS, every term except $\Delta \mathbf{v}_i$ is introduced due to using implicit integration
- It produces implicit filtering effect

Desbrun, M., Schröder, P., and Barr, A. 1999.
Interactive animation of structured deformable objects.
In *Proceedings of the 1999 Conference on Graphics interface '99*

If 2nd Term of $\partial \mathbf{f}_i / \partial \mathbf{x}_j$ is Omitted...

$$LHS = \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j$$

$$= \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) (\Delta \mathbf{v}_i^p + \Delta \mathbf{v}_i^o) - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} (\Delta \mathbf{v}_j^p + \Delta \mathbf{v}_j^o)$$

$$\Rightarrow \Delta \mathbf{v}_i + \left(\sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i^p - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j^p$$

$$RHS = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right)$$

$$= h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} (\mathbf{v}_{0j}^p + \mathbf{v}_{0j}^o) \right) \Rightarrow h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j}^p \right)$$

So, What is the Effect?

- The force in the orthogonal direction is not affected by implicit filtering.

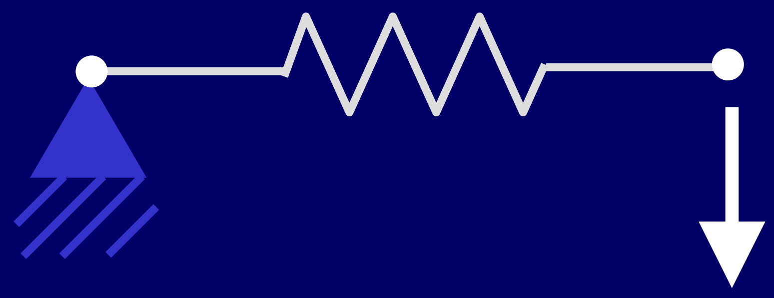
- Thus, the force in the orthogonal direction is in fact integrated by explicit method

What happens to Stability?

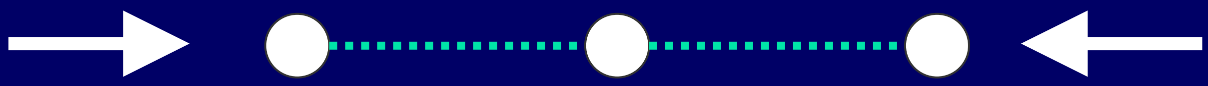
- By the omission, the system matrix is positive definite

- However, since the method is now not fully semi-implicit, stability is not guaranteed
  - If the orthogonal force is small compared to stretch force, there is little possibility system will diverge
  - How large is the orthogonal force?

When is the Ortho Force Generated?

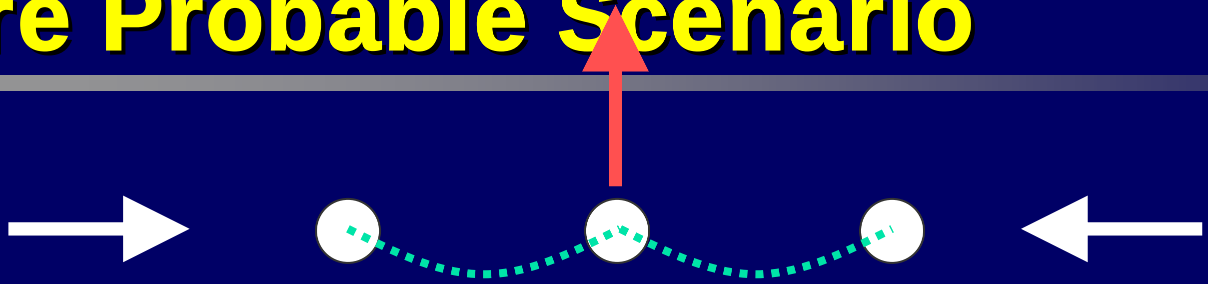


More Probable Scenario



- When a compressive force is applied to two consecutive links
  - Each link will immediately start to bend
  - However, at the center, an orthogonal force may be generated due to this compression

More Probable Scenario



- When a compressive force is applied to two consecutive links
  - Each link will immediately start to bend
  - However, at the center, an orthogonal force may be generated due to this compression
  - The force can be large when the mesh resolution is <u>high</u> (high res. is bad!)
  - The force can be large when the material's stiffness is large

Is there any other Scenario?

- We are not sure
- But based on our experiments, previous case is probably the most probable scenario that can possibly produce divergence.

Stability Analysis of Immediate Buckling Model

We cannot say it is unconditionally stable.

Artificial cases can be set up to create instabilities

But we did not meet any instabilities in practical cases

Physical Model of Cloth II

Kwang-Jin Choi

FXGear Inc.

Physical Model of Cloth II

Analysis on Damping used in Clothing Simulation

How Damping Affects Cloth Animation?

No damping

With some damping

Excessive damping

When Implicit Integration is Used…

$$\left(\mathbf{M} - \Delta t\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\frac{\Delta \mathbf{v}}{\Delta t} = \left(\mathbf{f}_0 + \Delta t\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0\right) \quad \text{(implcit)}$$

$$\mathbf{M}\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{f}_0, \quad \text{(explicit)}$$

Implicit Filtering

Artificial Damping Force

It creates artificial damping.

Desbrun, M., Schröder, P., and Barr, A. 1999.
Interactive animation of structured deformable objects.
In *Proceedings of the 1999 Conference on Graphics interface '99*

Results with Artificial Damping

$\Delta t = {}^{1}\!/_{90}\ \text{sec}$

- AD does not hurt the simulation very much
  - Even 1/30 shows no excessive damping
- Typically used time step size
  - 1/48 ~ 1/300
  - Because of collision resolution
- In most cases, extra damping needs to be added

Viscous Damping (Drag Force)

$$\mathbf{f} += -k_d \mathbf{v}$$

$$\left(\mathbf{M} + \Delta t k_d \mathbf{I} - \Delta t^2 \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right) \frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{f}_0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}_0$$

Push all the eigen values toward positive direction

Condition number gets closer to 1

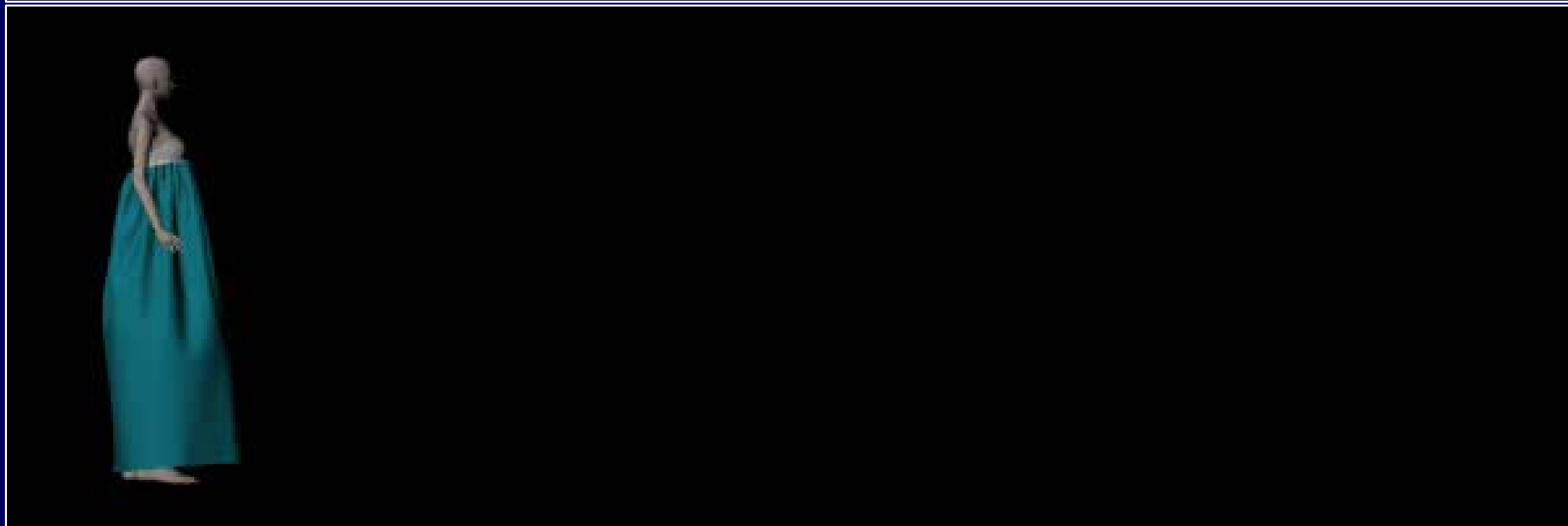
→ Faster convergence

Viscous Damping (Drag Force)

$$\mathbf{f} \mathrel{+}= -k_d \mathbf{v}$$

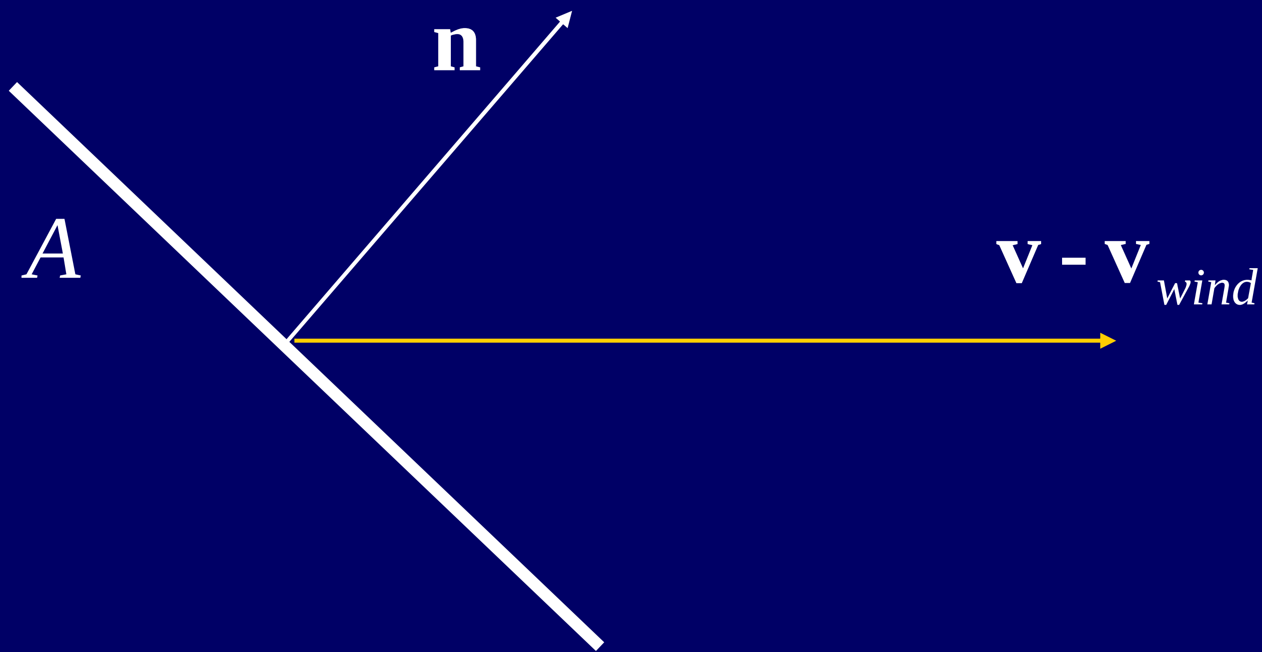
Drag

No drag



A Better Drag Force (Air Drag)

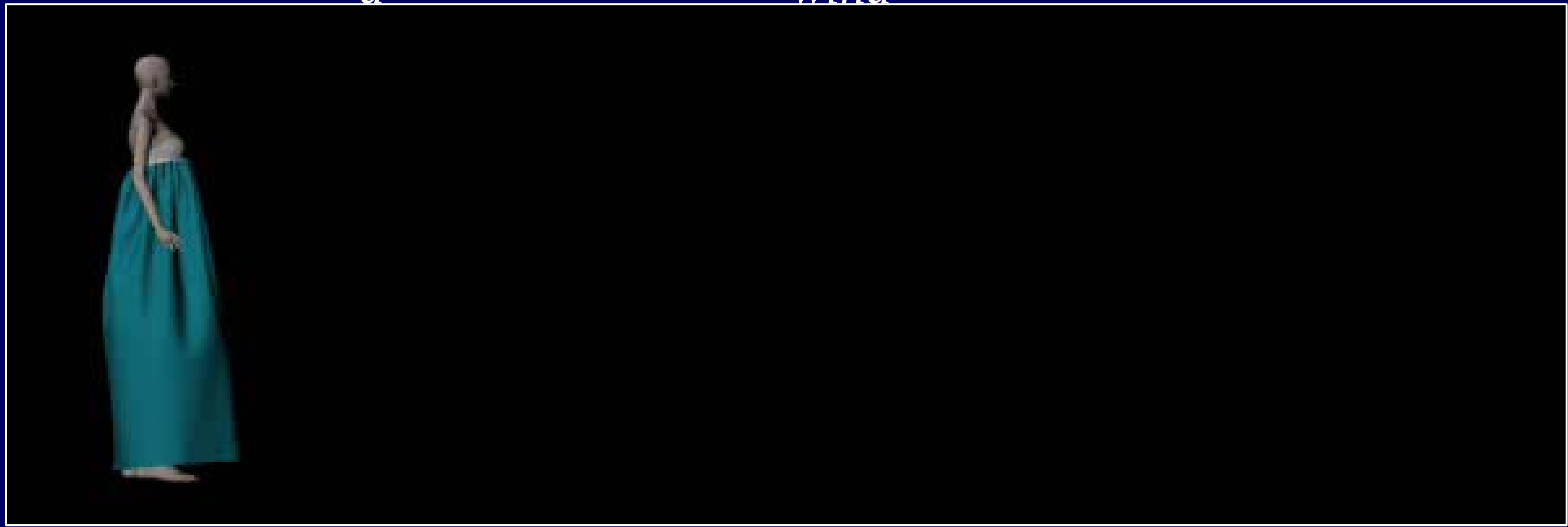
$$\mathbf{f} = -k_d A(\mathbf{n}^T(\mathbf{v} - \mathbf{v}_{wind}))\mathbf{n}$$



A Better Drag Force (Air Drag)

$$\mathbf{f} = -k_d A(\mathbf{n}^T(\mathbf{v} - \mathbf{v}_{wind}))\mathbf{n}$$

Air Drag

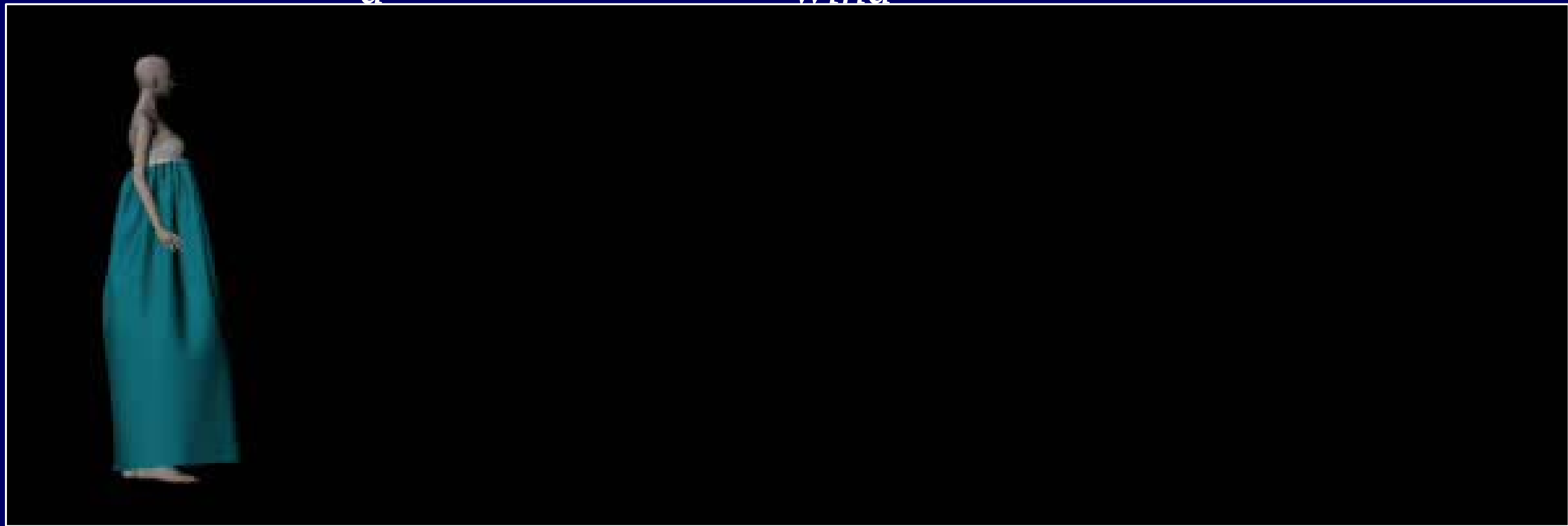


Viscous Drag

A Better Drag Force (Air Drag)

$$\mathbf{f} = -k_d A(\mathbf{n}^T(\mathbf{v} - \mathbf{v}_{wind}))\mathbf{n}$$

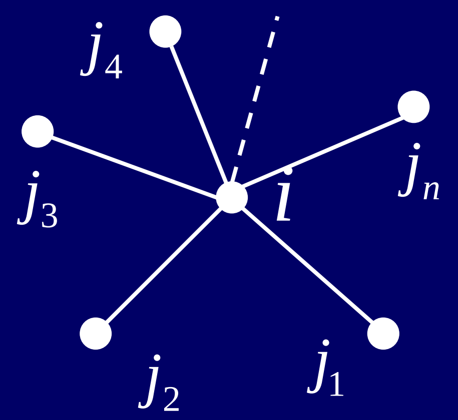
Air Drag



No Drag

Deformation Rate Damping

- Damp the deforming motion, not the rigid motion



$$\mathbf{f}_{ij} = -k_d(\mathbf{v}_i - \mathbf{v}_j)$$

$$\mathbf{f}_i = \sum_{j \in N} \mathbf{f}_{ij}$$

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{v}_j} = k_d \mathbf{I}$$

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{v}_i} = -k_d n \mathbf{I}$$

Deformation Rate Damping

$$\mathbf{f}_{ij} = -k_d(\mathbf{v}_i - \mathbf{v}_j)$$

$$\left(\mathbf{M} - \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right) \frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{f}_0 + \Delta t \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}_0$$

$\frac{\partial \mathbf{f}}{\partial \mathbf{v}}$ has zero eigenvalue for rigid translation

$\Rightarrow$ no dragging

Comparison to Drag Force

$$\mathbf{f}_{ij} = -k_d(\mathbf{v}_i - \mathbf{v}_j)$$

$$\mathbf{f}_i = -k_d\mathbf{v}_i$$

Comparison with Different k_d's

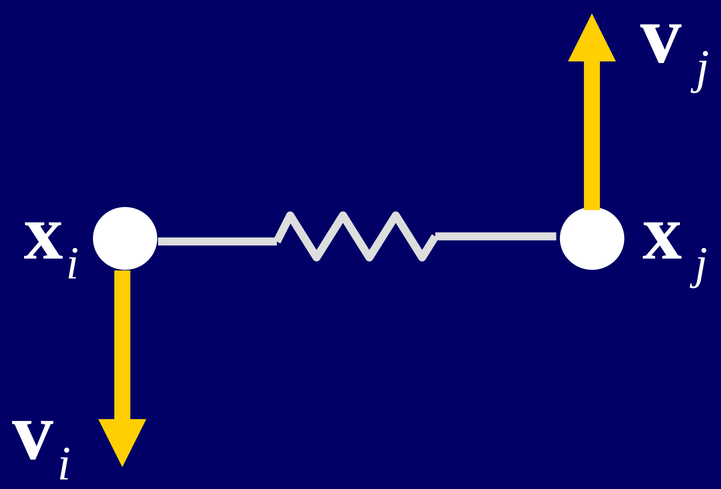
$$\mathbf{f}_{ij} = -k_d(\mathbf{v}_i - \mathbf{v}_j) \quad \mathbf{f}_{ij} = -k_d(\mathbf{v}_i - \mathbf{v}_j)$$

small k_d

large k_d

What about Rotation?

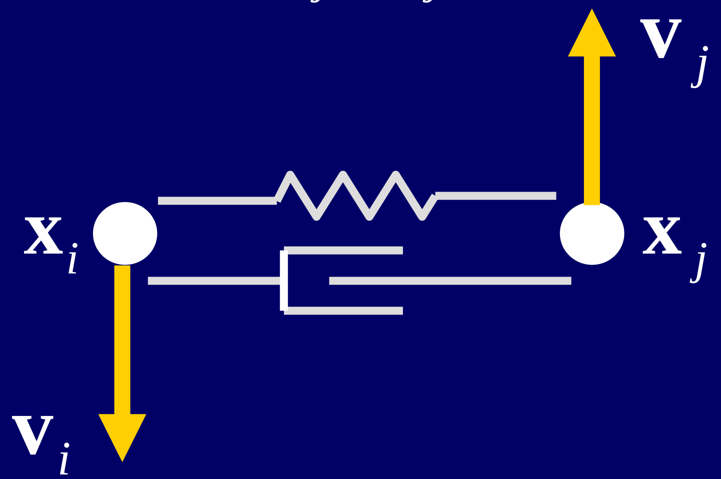
$$\mathbf{f}_{ij} = -k_d(\mathbf{v}_i - \mathbf{v}_j)$$



- The above force damps all the relative motions between particle i and j
- We want to damp only the motion in spring direction (x_i-x_j)

Damped Spring

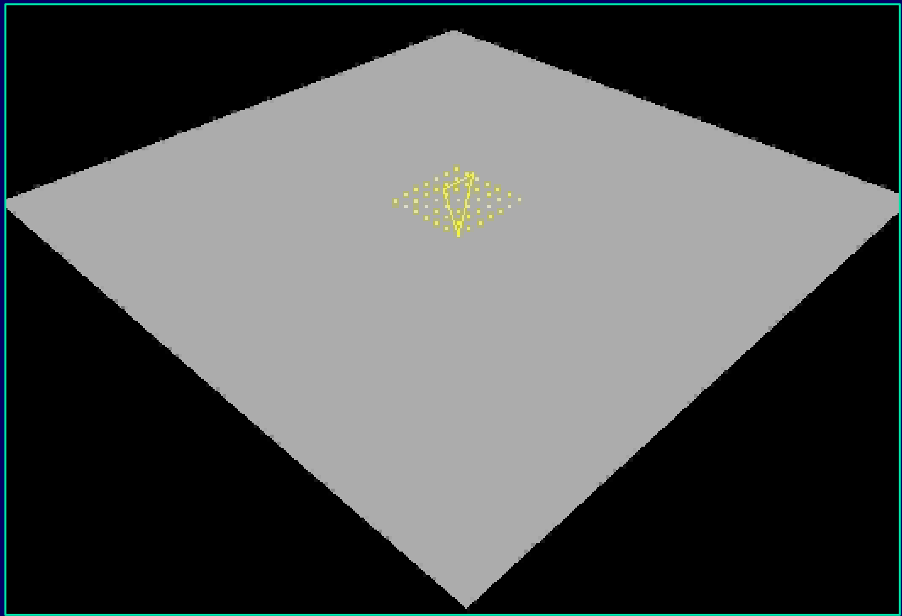
$$\mathbf{f}_{ij} = -k_d \frac{\mathbf{x}_{ij}\mathbf{x}_{ij}^T}{\mathbf{x}_{ij}^T\mathbf{x}_{ij}}(\mathbf{v}_i - \mathbf{v}_j)$$

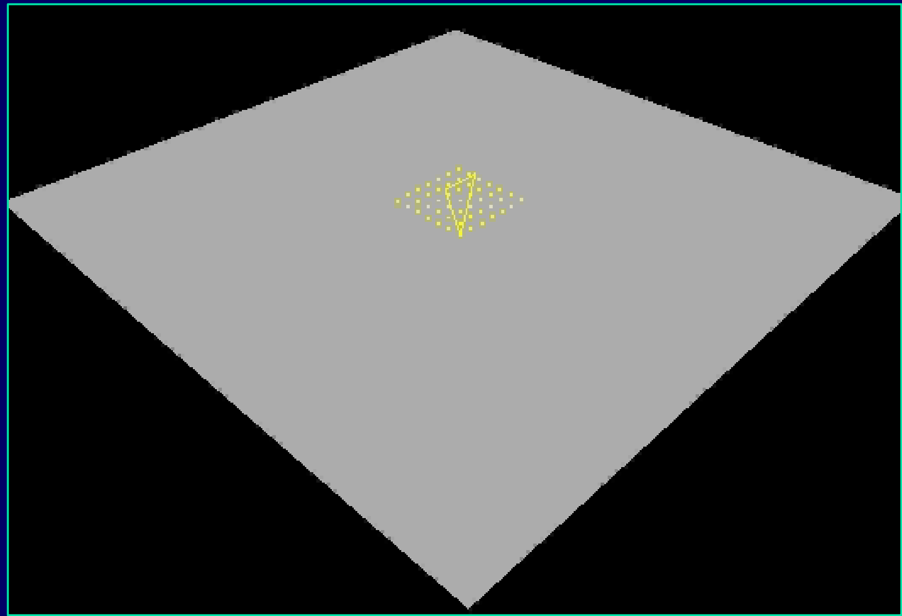


- Minimal damping to rotation
- Damping to bending and stretching can be decoupled

Comparison

$$\mathbf{f}_{ij} = -k_d(\mathbf{v}_i - \mathbf{v}_j) \qquad \mathbf{f}_{ij} = -k_d \frac{\mathbf{x}_{ij}\mathbf{x}_{ij}^T}{\mathbf{x}_{ij}^T\mathbf{x}_{ij}}(\mathbf{v}_i - \mathbf{v}_j)$$





More General Deformation Rate Damping

- Define deformation $$D = (|\mathbf{x}_i - \mathbf{x}_j| - L) = (|\mathbf{x}_{ij}| - L)$$
- Get the deforming direction $$\frac{\partial D}{\partial \mathbf{x}_{ij}} = \frac{\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|}$$
- Get the deformation rate $$\dot{D} = \left(\frac{\partial D}{\partial \mathbf{x}_{ij}}\right)^T \frac{d\mathbf{x}_{ij}}{dt} = \frac{\mathbf{x}_{ij}^T}{|\mathbf{x}_{ij}|}\mathbf{v}_{ij}$$
- Damp the deformation to the deforming direction

$$\mathbf{f}_{ij} = -k_d \frac{\partial D}{\partial \mathbf{x}_{ij}} \dot{D} = -k_d \frac{\partial D}{\partial \mathbf{x}_{ij}} \left(\frac{\partial D}{\partial \mathbf{x}_{ij}}\right)^T \mathbf{v}_{ij} = -k_d \frac{\mathbf{x}_{ij}\mathbf{x}_{ij}^T}{|\mathbf{x}_{ij}|^2}\mathbf{v}_{ij}$$

Baraff and Witkin, *Large Steps in Cloth Simulation*, SIGGRAPH '98

Putting All Together

$$\mathbf{f}^{damp} = -k_d \sum_i \left(\frac{\partial c^i}{\partial \mathbf{x}} \left(\frac{\partial c^i}{\partial \mathbf{x}} \right)^T \mathbf{v} \right) - k_a \mathbf{n}\mathbf{n}^T (\mathbf{v} - \mathbf{v}^{wind})$$

$$\frac{\partial \mathbf{f}^{damp}}{\partial \mathbf{v}} = -k_d \sum_i \frac{\partial c^i}{\partial \mathbf{x}} \left(\frac{\partial c^i}{\partial \mathbf{x}} \right)^T - k_a \mathbf{n}\mathbf{n}^T$$

Putting All Together

Course Schedule

1st Session

1:45~2:00 Introduction (Ko)

2:00~3:00 Physical Model of Cloth I (Ko)

3:00~3:30 Physical Model of Cloth II (Choi)

2nd Session

3:45~4:30 Collision Handling (Bridson)

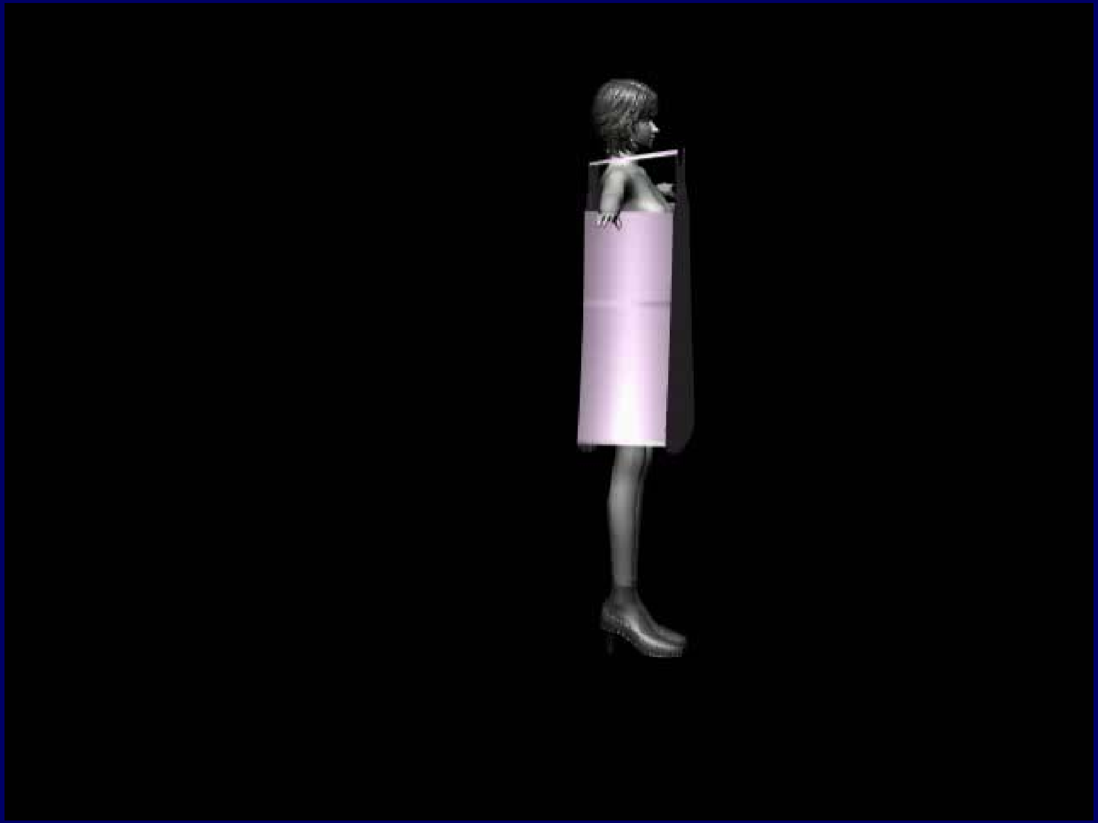
4:30~5:00 Cloth Design and Applications (Zhang)

5:00~5:30 Current State-of-the Art / Challenges Ahead (Ko)

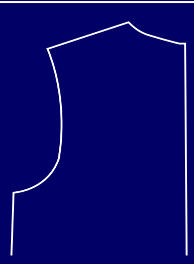
Current State-of-the-Art and Challenges Ahead

Hyeong-Seok Ko
Seoul National Univ.
Graphics & Media Lab

Would it be Possible?



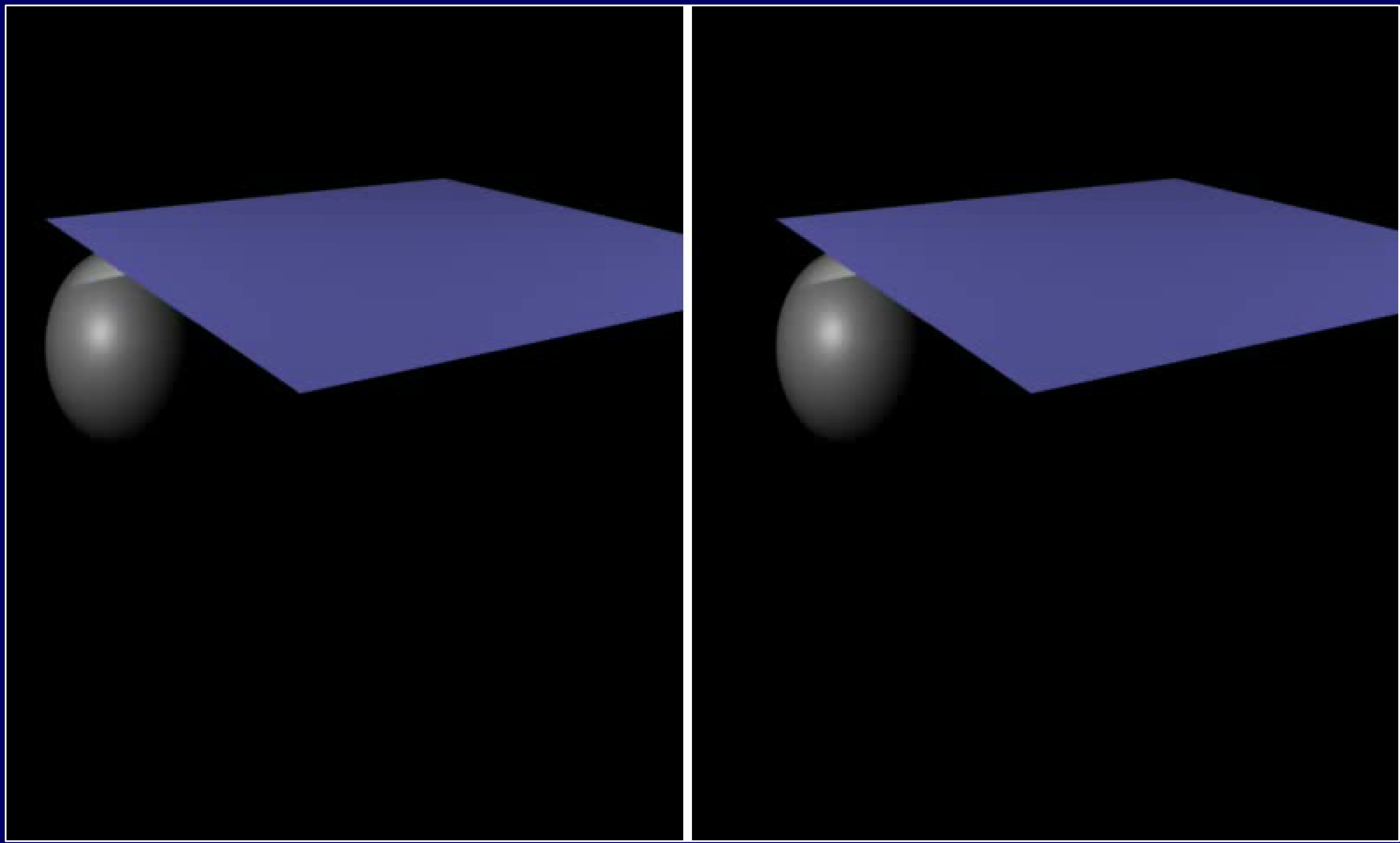
Source of Frustrations



- Poor collision handling
- Allowed only regular, rectangular grids
  - Not practical for creating complex garments
- Garments had to be constructed from 2D patterns
  - Not practical for 3D animation purpose
- Few Controls

Collision Handling 2005

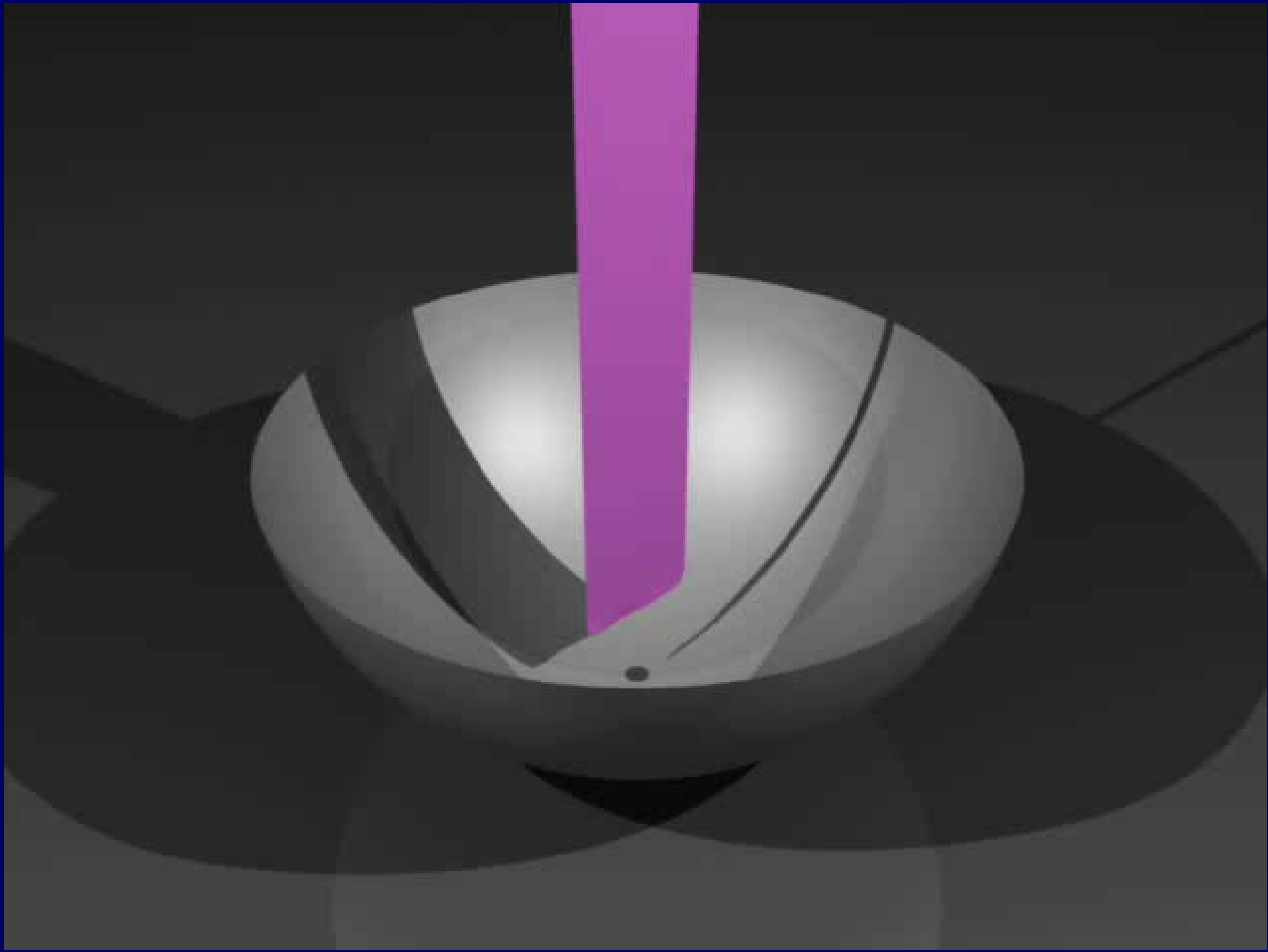
Collision Test



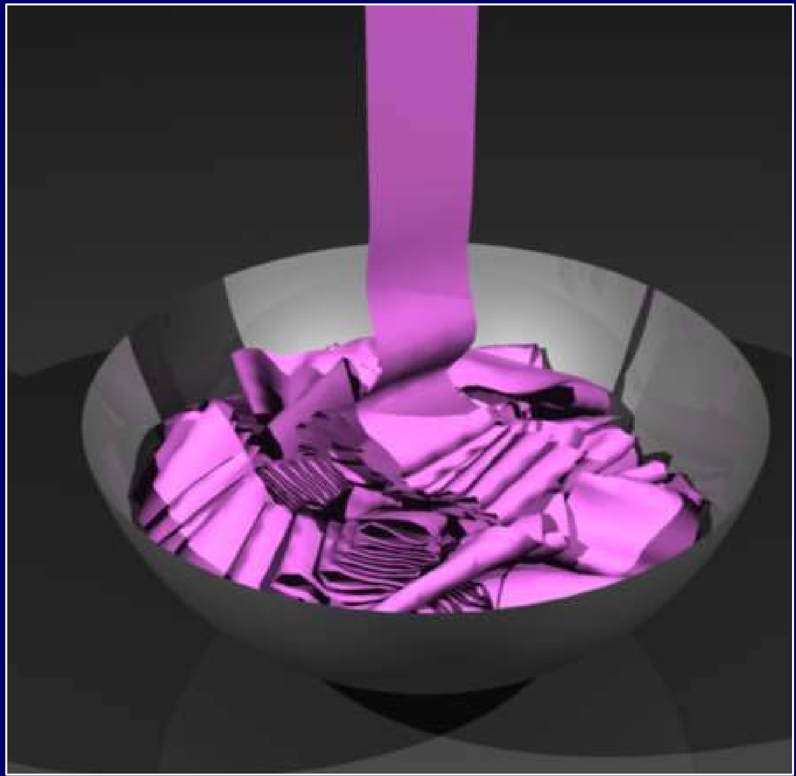
Collision Test

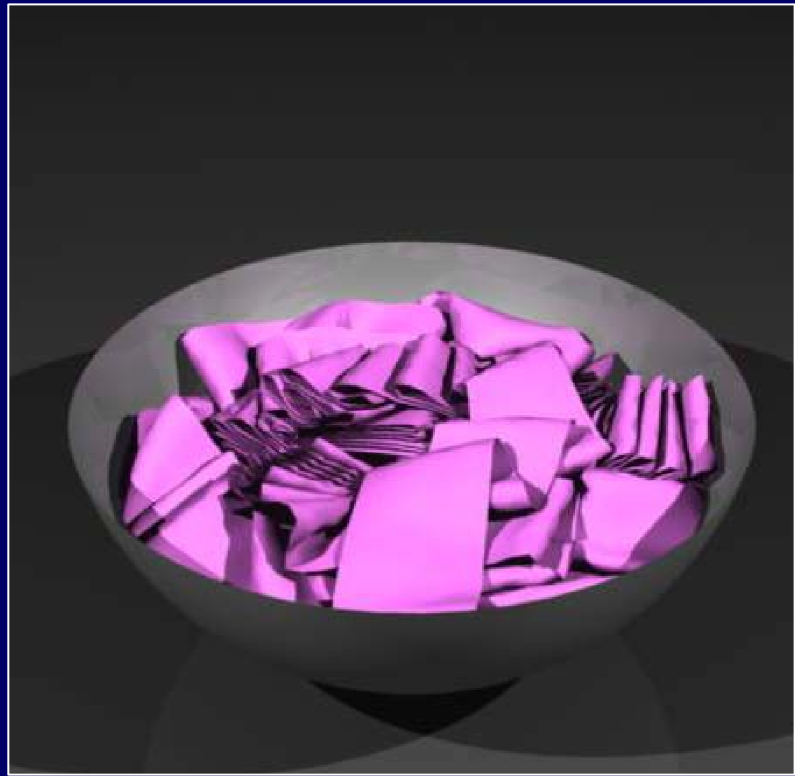
# of vertices	Animation Length	Computation Time
6029	20 sec	31.5 mins

Self-Collision Test



Simulation Statistics



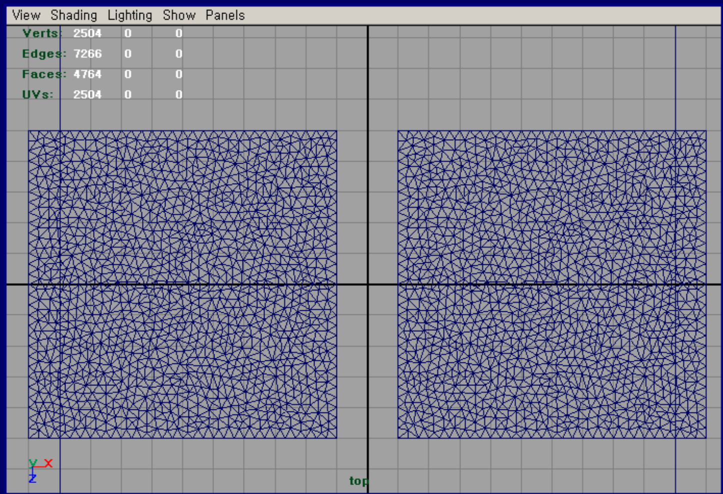


- 0.1m X 50m cloth
- 28,007 vertices
- 15 hours

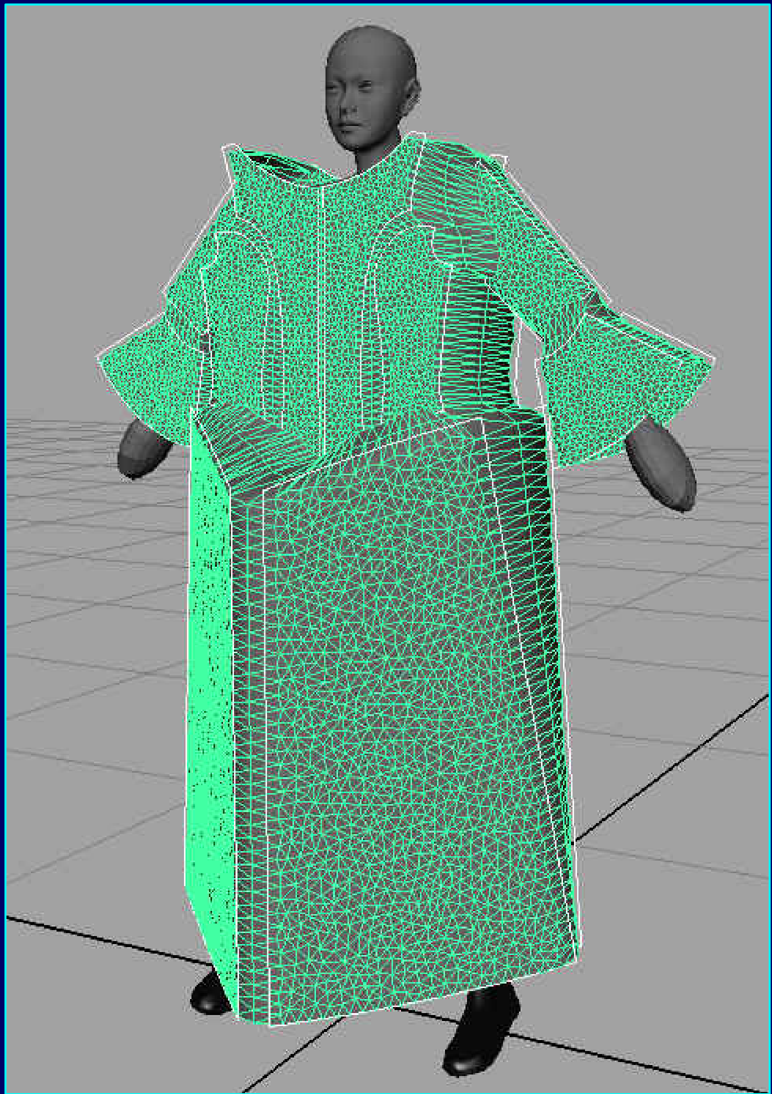
Collision Handling in 3 Pieces

Garment Construction 2005

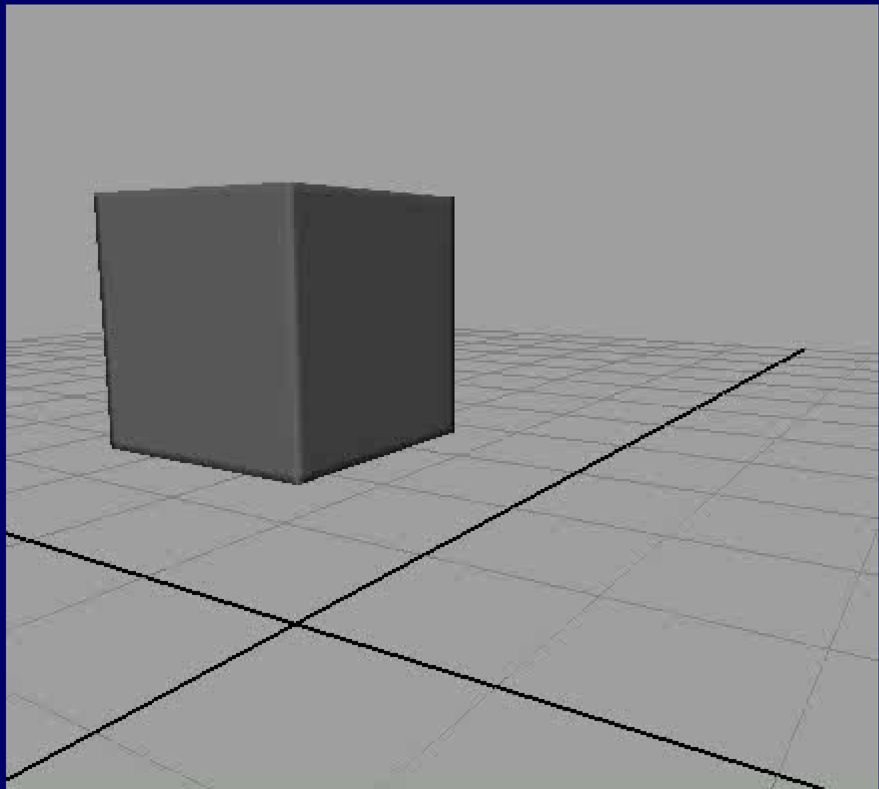
Unstructured Triangular Mesh



Construction from 2D Patterns

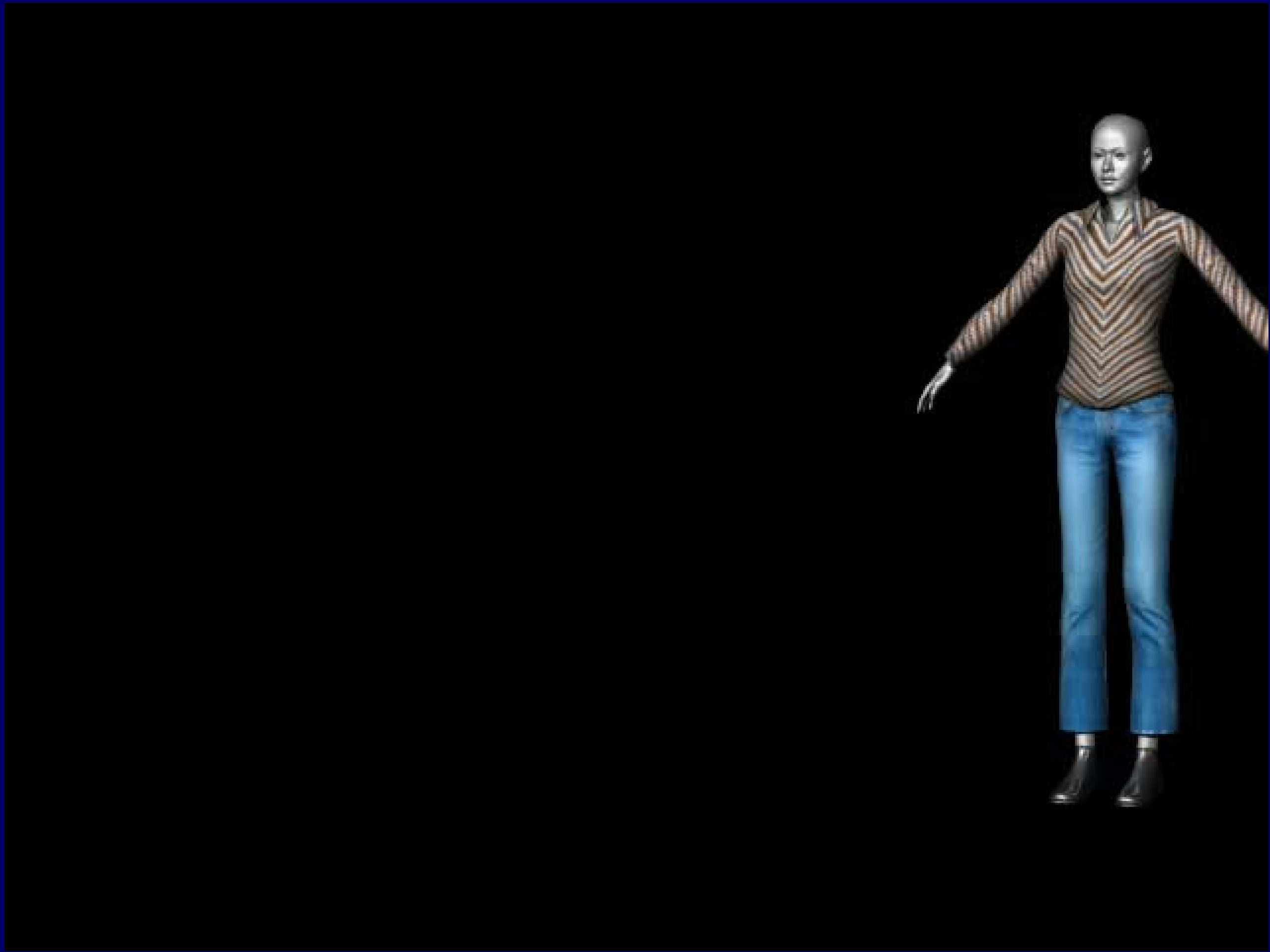


Construction from 3D Modeling



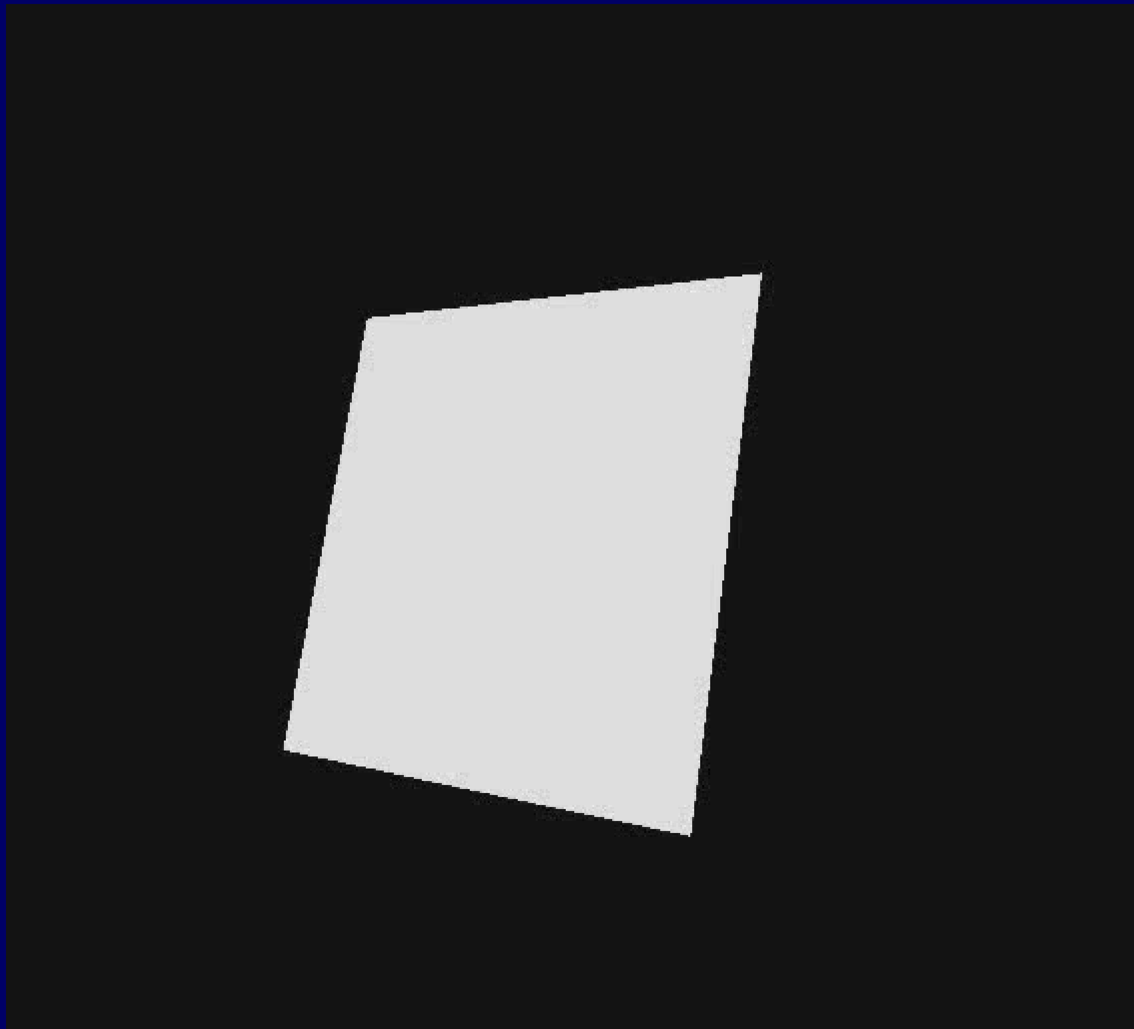
Construction from 3D Modeling

Construction from 3D Modeling



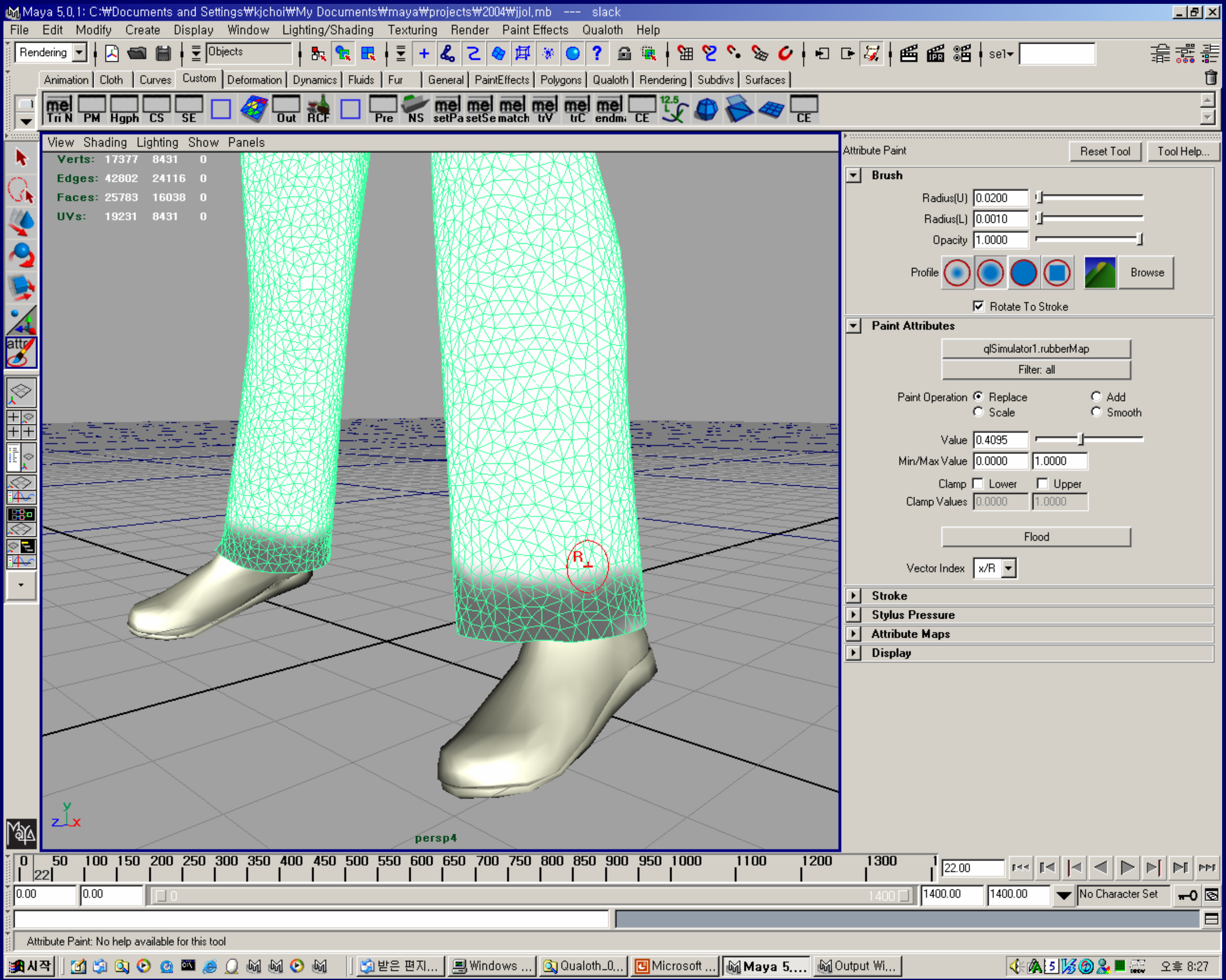
Other Controls 2005

Wrinkle Map



AXIS Qualoth

Rubber Map



Maya 5.0.1: C:\Documents and Settings\kjchoi\My Documents\maya\projects\2004\jjol.mb --- slack
File Edit Modify Create Display Window Lighting/Shading Texturing Render Paint Effects Qualoth Help
Animation Cloth Curves Custom Deformation Dynamics Fluids Fur General PaintEffects Polygons Qualoth Rendering Subdivs Surfaces
View Shading Lighting Show Panels
Verts: 17377 8431 0
Edges: 42802 24116 0
Faces: 25783 16038 0
UVs: 19231 8431 0
persp4
Attribute Paint
Reset Tool
Tool Help...
Brush
Radius(U) 0.0200
Radius(L) 0.0010
Opacity 1.0000
Profile
Browse
Rotate To Stroke
Paint Attributes
qlSimulator1.rubberMap
Filter: all
Paint Operation Replace Add Scale Smooth
Value 0.4095
Min/Max Value 0.0000 1.0000
Clamp Lower Upper
Clamp Values 0.0000 1.0000
Flood
Vector Index x/R
Stroke
Stylus Pressure
Attribute Maps
Display
No Character Set
Attribute Paint: No help available for this tool

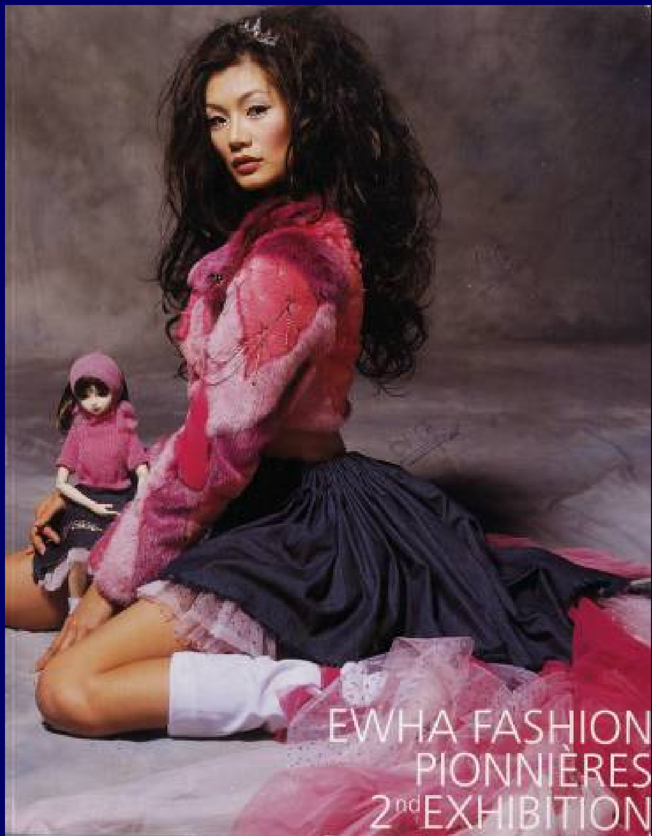
Rubber Map

Effect of Buttons

Wind Fields

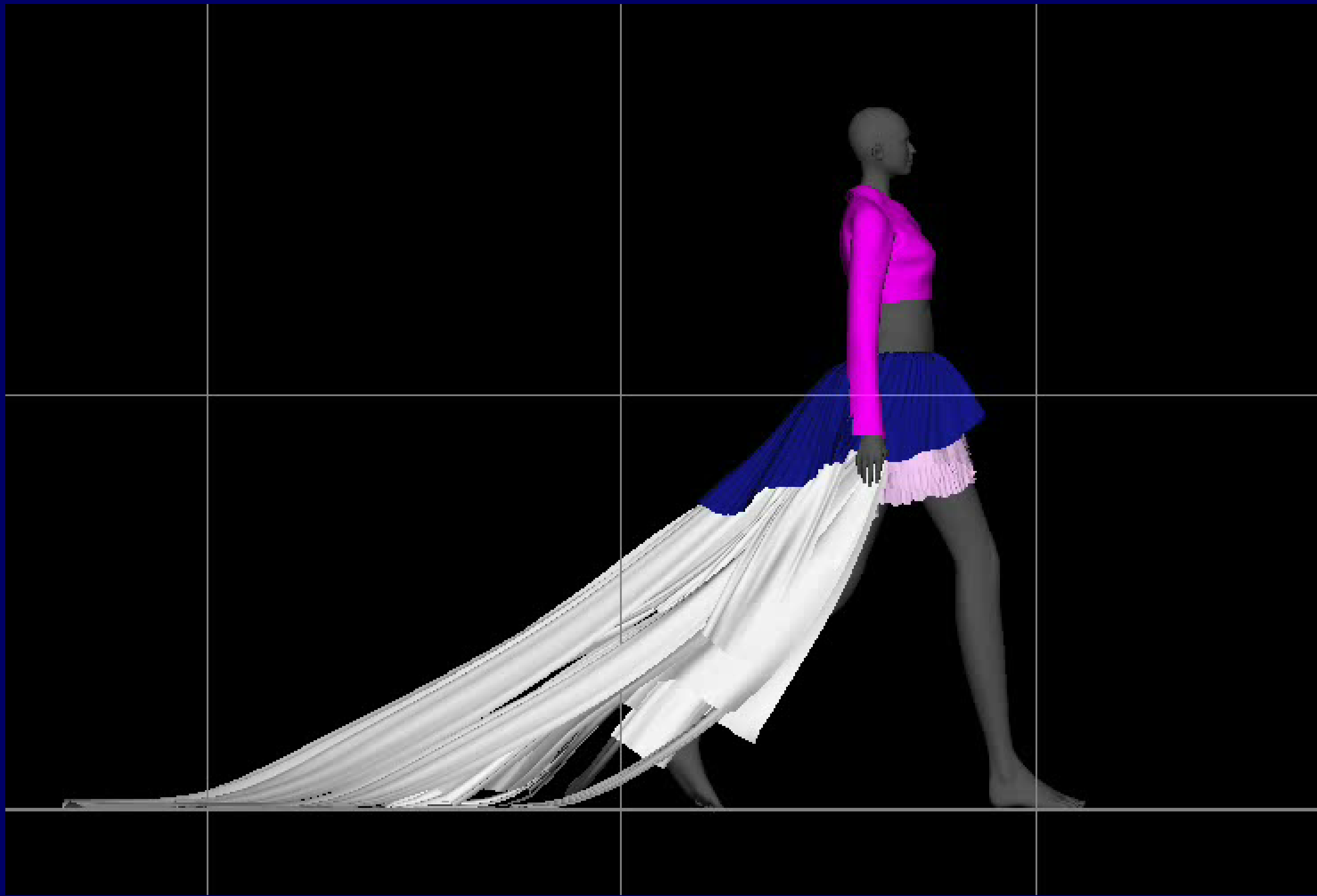
How Far can it Go?

Reconstruction of the Clothes

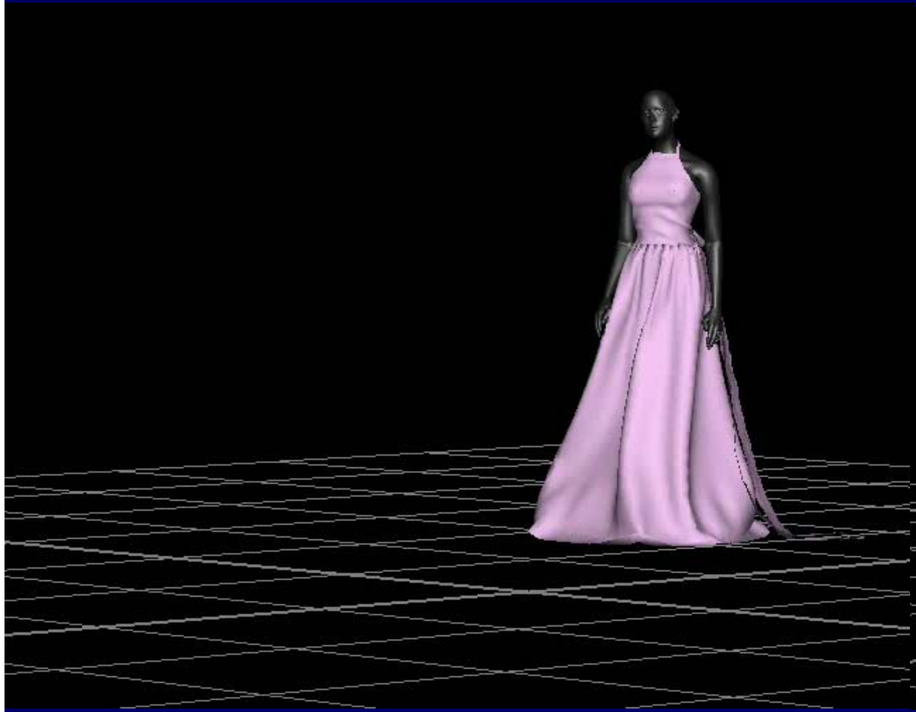


Reconstruction of the Clothes

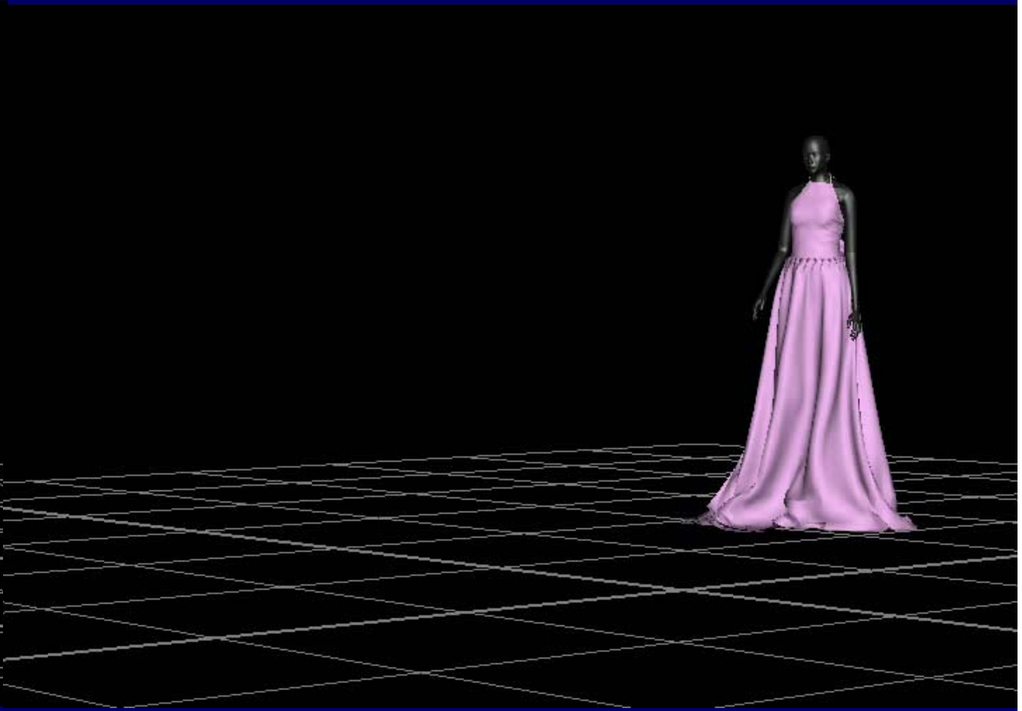
The Simulation



The Simulation



Original Stiffness



Intentional Modification

Now, We may call it a “Fashion”

Challenges Ahead

- Physical Model and Simulation
  - Modeling nonlinear and hysteretic properties
  - Exploring the continuum approach
  - Increasing the fidelity of simulated cloth
  - Increasing algorithm speed while maintaining reasonable quality
- Collision Resolution
  - Rapid collision detection
  - Accurate collision detection
  - Robust collision response generation

Thank You